syntax, main operator

Rules of Syntax for SL

Any expression of SL that follows the following rules is an allowable expression of SL. Such allowable expressions are called WFFs (Well-Formed Formulae)

Rules of Syntax for SL

Rule 0 (not in text but should be):

An atomic statement is a WFF

Rule 0.1 (not in the text but should be):

For every expression in SL with more than once occurrence of an atomic statement, each atomic statement must be in the scope of at least one connective.

Rule 0.2 (you get the picture)

A WFF can be in the scope of only one logical operator (~, v, ·, ⊃, and ≡)

Rules of Syntax for SL

Negation Rule:

A negation may only be placed at the left of a WFF.

Only the WFF to the right of the negation is in its scope.

Rules of Syntax for SL

Rule for all other connectives:

The connectives (v, $\cdot$, $\supset$, and $\equiv$) may only be placed between two WFFs.

Only the WFFs immediately to either side of the connectives v, $\cdot$, $\supset$, and $\equiv$ are in the scope of the connective.

Rules of Syntax for SL

Rule for parentheses and brackets:

A pair of parentheses or brackets may only be placed around a WFF.

Parentheses and brackets must be used to clarify any instance of scope ambiguity.

The Main Connective

- For the purposes of truth tables, it is critical to be able to identify what is in the scope of each connective in an expression.
- The connective that has an entire WFF in its scope is called the ‘main connective’.

An Example: Building a WFF

S

This is a WFF because it follows Rule 0

There is no main connective because there are no connectives

An Example: Building a WFF

$R \supset S$

Main connective

This is a WFF because it connects two WFFs following the rule for connectives.

An Example: Building a WFF

$R \supset S \vee P$

Main Connective?

This is not a WFF because S is in the scope of both the conditional and the disjunction (Rule 0.2). It requires parentheses to clarify.

An Example: Building a WFF

$(R \supset S) \vee P$

Main Connective

Now the WFF ‘S’ is in the scope of the conditional, while the WFF ‘$R \supset S$’ is in the scope of the disjunction. The above expression is now a WFF.

An Example: Building a WFF

$(R \supset S) \vee \sim P$

Main Connective

This is a WFF, and will remain so as we build it out.

An Example: Building a WFF

$[P \cdot (R \supset S)] \vee \sim P$

Main Connective

An Example: Building a WFF

$$[(P \vee \sim Q) \cdot (R \supset S)] \vee \sim P$$

Main Connective

An Example: Building a WFF

$$([(P \vee \sim Q) \cdot (R \supset S)] \vee \sim P) \supset R$$

Main Connective

An Example: Building a WFF

$$([(P \vee \sim Q) \cdot (R \supset S)] \vee \sim P) \supset (R \equiv M)$$

Main connective

An Example: Building a WFF

$$([(P \vee \sim Q) \cdot (R \supset S)] \vee \sim P) \supset \sim(R \equiv M)$$

Main Connective

This expression is a WFF and it is a conditional.
The antecedent is a disjunction

The consequent is a negation