

Exercises 5.1.1E

1.a. Derive: $A \& B$

1	A	Assumption
2	A \supset B	Assumption
3	B	1, 2 \supset E
4	A & B	1, 3 &I

c. Derive: $A \supset (\sim C \& \sim B)$

1	A \supset ($\sim B \& \sim C$)	Assumption
2	A	A / \supset I
3	$\sim B \& \sim C$	1, 2 \supset E
4	$\sim B$	3 &E
5	$\sim C$	3 &E
6	$\sim C \& \sim B$	4, 5 &I
7	A \supset ($\sim C \& \sim B$)	1-6 \supset I

e. Derive: $\sim A \supset [B \& (D \& C)]$

1	$\sim A \supset B$	Assumption
2	B \supset D	Assumption
3	$\sim A \supset C$	Assumption
4	$\sim A$	A / \supset I
5	B	1, 4 \supset E
6	D	2, 5 \supset E
7	C	3, 6 \supset E
8	D & C	6, 7 &I
9	B & (D & C)	5, 8 &I
10	$\sim A \supset [B \& (D \& C)]$	4-9 \supset I

g. Derive: $[(K \vee L) \supset I] \& [(K \vee L) \supset \sim J]$

1	(K \vee L) \supset (I & \sim J)	Assumption
2	K \vee L	A / \supset I
3	I & \sim J	1, 2 \supset E
4	I	3 &E
5	(K \vee L) \supset I	2-4 \supset I
6	K \vee L	A / \supset I
7	I & \sim J	1, 6 \supset E
8	\sim J	7 &E
9	(K \vee L) \supset \sim J	6-8 \supset E
10	[(K \vee L) \supset I] & [(K \vee L) \supset \sim J]	5, 9 &I

i. Derive: $A \supset (B \supset C)$

1	(A & B) \supset C	Assumption
2	A	A / \supset I
3	B	A / \supset I
4	A & B	2, 3 & I
5	C	1, 4 \supset E
6	B \supset C	3-5 \supset I
7	A \supset (B \supset C)	2-6 \supset I

k. Derive: $(A \& B) \supset (C \& D)$

1	(B & A) \supset (D & C)	Assumption
2	A & B	A / \supset I
3	B	2 &E
4	A	2 &E
5	B & A	3, 4 &I
6	D & C	1, 5 \supset E
7	C	6 &E
8	D	6 &E
9	C & D	7, 8 &I
10	(A & B) \supset (C & D)	2-9 \supset I

m. Derive: $(A \& B) \supset E$

1	A \supset C	Assumption
2	B \supset D	Assumption
3	(C & D) \supset E	Assumption
4	A & B	A / \supset I
5	A	4 &E
6	B	4 &E
7	C	1, 5 \supset E
8	D	2, 6 \supset E
9	C & D	7, 8 &I
10	E	3, 9 \supset E
11	(A & B) \supset E	4-10 \supset I

Exercises 5.1.2E

1.a. Derive: $\sim G$

1	(G \supset I) & \sim I	Assumption
2	G	A / \sim I
3	G \supset I	1 &E
4	I	2, 3 \supset E
5	\sim I	1 &E
6	\sim G	2-5 \sim I

c. Derive: $\sim \sim B$

1	$\sim B \supset A$	Assumption		
2	$\sim B \supset \sim A$	Assumption		
3	<table style="border-collapse: collapse;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">$\sim B$</td> <td style="padding-left: 20px;">A / \sim I</td> </tr> </table>	$\sim B$	A / \sim I	
$\sim B$	A / \sim I			
4	<table style="border-collapse: collapse;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">A</td> <td style="padding-left: 20px;">1, 3 \supsetE</td> </tr> </table>	A	1, 3 \supset E	
A	1, 3 \supset E			
5	$\sim A$	2, 3 \supset E		
6	$\sim \sim B$	3-5 \sim I		

e. Derive: A

1	$(\sim A \supset \sim B) \& (\sim B \supset B)$	Assumption		
2	<table style="border-collapse: collapse;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">$\sim A$</td> <td style="padding-left: 20px;">A / \sim E</td> </tr> </table>	$\sim A$	A / \sim E	
$\sim A$	A / \sim E			
3	$\sim A \supset \sim B$	1 &E		
4	$\sim B$	2, 3 \supset E		
5	$\sim B \supset B$	1 &E		
6	B	4, 5 \supset E		
7	A	2-6 \sim E		

Exercises 5.1.3E

1.a. Derive: $B \vee (K \vee G)$

1	K	Assumption
2	$K \vee G$	I \vee I
3	$B \vee (K \vee G)$	2 \vee I

c. Derive: $D \vee E$

1	$E \vee D$	Assumption		
2	<table style="border-collapse: collapse;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">E</td> <td style="padding-left: 20px;">A / \veeE</td> </tr> </table>	E	A / \vee E	
E	A / \vee E			
3	$D \vee E$	2 \vee I		
4	<table style="border-collapse: collapse;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">D</td> <td style="padding-left: 20px;">A / \veeE</td> </tr> </table>	D	A / \vee E	
D	A / \vee E			
5	$D \vee E$	4 \vee I		
6	$D \vee E$	1, 2-3, 4-5 \vee E		

e. Derive: F

1	$\sim E \vee F$	Assumption		
2	$\sim E \supset F$	Assumption		
3	<table style="border-collapse: collapse;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">$\sim E$</td> <td style="padding-left: 20px;">A / \veeE</td> </tr> </table>	$\sim E$	A / \vee E	
$\sim E$	A / \vee E			
4	F	2, 3 \supset E		
5	<table style="border-collapse: collapse;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">F</td> <td style="padding-left: 20px;">A / \veeE</td> </tr> </table>	F	A / \vee E	
F	A / \vee E			
6	F	5 R		
7	F	1, 3-4, 5-6 \vee E		

Exercises 5.1.4E

1.a. Derive: L

1	K \equiv (\sim E & L)	Assumption
2	K	Assumption
3	\sim E & L	1, 2 \equiv E
4	L	3 &E

c. Derive: S & \sim A

1	(S \equiv \sim I) & N	Assumption
2	(N \equiv \sim I) & \sim A	Assumption
3	\sim A	2 &E
4	N \equiv \sim I	2 &E
5	N	1 &E
6	\sim I	4, 5 \equiv E
7	S \equiv \sim I	1 &E
8	S	6, 7 \equiv E
9	S & \sim A	3, 8 &I

e. Derive: E \equiv O

1	(E \supset T) & (T \supset O)	Assumption
2	O \supset E	Assumption
3	E	A / \equiv I
4	E \supset T	1 &E
5	T	3, 4 \supset E
6	T \supset O	1 &E
7	O	5, 6 \supset E
8	O	A / \equiv I
9	E	2, 8 \supset E
10	E \equiv O	3-7, 8-9 \equiv I

Exercises 5.3E

1. Derivability

a. Derive: A \supset (A & B)

1	A \supset B	Assumption
G	A \supset (A & B)	2-__ \supset I

Derive: $A \supset (A \& B)$

1	A \supset B	Assumption
2	A	A / \supset I
3	B	1, 2 \supset E
4	A & B	2, 3 &I
5	A \supset (A & B)	2-5 \supset I

c. Derive: $L \equiv K$

1	(K \supset L) & (L \supset K)	Assumption
2	L	A / \equiv I
G	K	
G	K	A / \equiv I
G	L	
G	L \equiv K	2-___, ___-___ \equiv I

Derive: $L \equiv K$

1	(K \supset L) & (L \supset K)	Assumption
2	L	A / \equiv I
3	L \supset K	1 &E
4	K	2, 3 \supset E
5	K	A / \equiv I
6	K \supset L	1 &E
7	L	5, 6 \supset E
8	L \equiv K	2-4, 5-7 \equiv I

e. Derive: C

1	B & \sim B	Assumption
2	\sim C	A / \sim E
G	C	2-___ \sim E

Derive: C

1	B & ~ B	Assumption
2	~ C	A /~ E
3	B	1 &E
4	~ B	1 &E
5	C	2-4 ~ E

g. Derive: $D \supset B$

1	$A \supset C$	Assumption
2	$(\sim A \vee C) \supset (D \supset B)$	Assumption
G	$\sim A \vee C$	
G	$D \supset B$	2, \supset E

Derive: $D \supset B$

1	$A \supset C$	Assumption
2	$(\sim A \vee C) \supset (D \supset B)$	Assumption
3	$\sim (\sim A \vee C)$	A /~ E
4	A	A /~ I
5	C	1, 4 \supset E
6	$\sim A \vee C$	5 \vee I
7	$\sim (\sim A \vee C)$	3 R
8	$\sim A$	4-7 \sim I
9	$\sim A \vee C$	8 \vee I
10	$\sim (\sim A \vee C)$	3 R
11	$\sim A \vee C$	3, 10 \sim E
12	$D \supset B$	2, 11 \supset E

i. Derive: B

1	A \supset B	Assumption
2	\sim (B & \sim C) \supset A	Assumption
3	<div style="border-left: 1px solid black; padding-left: 10px; margin-left: 10px;">\sim B</div>	A / \sim E
G	B	3- \sim E

Derive: B

1	A \supset B	Assumption
2	\sim (B & \sim C) \supset A	Assumption
3	<div style="border-left: 1px solid black; padding-left: 10px; margin-left: 10px;">\sim B</div>	A / \sim E
4	<div style="border-left: 1px solid black; padding-left: 10px; margin-left: 10px;">B & \sim C</div>	A / \sim I
5	<div style="border-left: 1px solid black; padding-left: 10px; margin-left: 10px;">B</div>	4 &E
6	<div style="border-left: 1px solid black; padding-left: 10px; margin-left: 10px;">\sim B</div>	3 R
7	\sim (B & \sim C)	4-6 \sim I
8	A	2, 7 \supset E
9	B	1, 8 \supset E
10	\sim B	3 R
11	B	3-10 \sim E

k. Derive: B \vee \sim C

1	A \vee (B & C)	Assumption
2	C \supset \sim A	Assumption
3	<div style="border-left: 1px solid black; padding-left: 10px; margin-left: 10px;">A</div>	A / \vee E
G	B \vee \sim C	
G	<div style="border-left: 1px solid black; padding-left: 10px; margin-left: 10px;">B & C</div>	A / \vee E
G	B \vee \sim C	
G	B \vee \sim C	1, 3- \sim , \sim - \vee E

Derive: $B \vee \sim C$

1	A \vee (B & C)		Assumption
2	C \supset \sim A		Assumption
3	A		A / \vee E
4	C		A / \sim I
5	\sim A		2, 4 \supset E
6	A		3 R
7	\sim C		4-6 \sim 1
8	B \vee \sim C		7 \vee I
9	B & C		A / \vee E
10	B		9 &E
11	B \vee \sim C		10 \vee I
12	B \vee \sim C		1, 3-8, 9-11 \vee E

m. Derive: $D \supset (F \supset C)$

1	(A \vee B) \supset C		
2	(D \vee E) \supset [(F \vee G) \supset A]		
3	D		A / \supset I
G	F \supset C		
G	D \supset (F \supset C)		3- 4 \supset I

Derive: $D \supset (F \supset C)$

1	(A \vee B) \supset C		
2	(D \vee E) \supset [(F \vee G) \supset A]		
3	D		A / \supset I
4	F		A / \supset I
5	D \vee E		3 \vee I
6	(F \vee G) \supset A		2, 5 \supset E
7	F \vee G		4 \vee I
8	A		6, 7 \supset E
9	A \vee B		8 \vee I
10	C		1, 9 \supset E
11	F \supset C		4-10 \supset I
12	D \supset (F \supset C)		3-11 \supset I

o. Derive: $B \supset F$

1	A \supset \sim (B \vee C)	Assumption
2	(C \vee D) \supset A	Assumption
3	\sim F \supset (D & \sim E)	Assumption
4	B	A / \supset I
G	F	
G	B \supset F	4— \supset I

Derive: $B \supset F$

1	A \supset \sim (B \vee C)	Assumption
2	(C \vee D) \supset A	Assumption
3	\sim F \supset (D & \sim E)	Assumption
4	B	A / \supset I
5	\sim F	A / \sim E
6	D & \sim E	3, 5 \supset E
7	D	6 &E
8	C \vee D	7 \vee I
9	A	2, 8 \supset E
10	\sim (B \vee C)	1, 9 \supset E
11	B \vee C	4 \vee I
12	F	5–11 \sim I
13	B \supset F	4–12 \supset I

q. Derive: H

1	F \supset (G \vee H)	Assumption
2	\sim (\sim F \vee H)	Assumption
3	\sim G	Assumption
4	\sim H	A / \sim E
G	\sim F \vee H	
G	\sim (\sim F \vee H)	2 R
G	H	4— \sim E

q. Derive: H

1	F \supset (G \vee H)		Assumption
2	\sim (\sim F \vee H)		Assumption
3	\sim G		Assumption
4	\sim H		A / \sim E
5	F		A / \sim I
6	G \vee H		1, 5 \supset E
7	G		A / \vee E
8	\sim H		A / \sim E
9	G		7 R
10	\sim G		3 R
12	H		8–10 \sim E
13	H		A / \vee E
14	\sim H		4 R
15	H		13 R
16	H		6, 7–12, 13–15 \vee E
17	\sim H		4 R
18	\sim F		5–17 \sim E
19	\sim F \vee H		18 \vee I
20	\sim (\sim F \vee H)		2 R
21	H		4–20 \sim E

2. Validity

a. Derive: A \supset C

1	A \supset \sim B		Assumption
2	\sim B \supset C		Assumption
3	A		A / \supset I
4	\sim B		1, 3 \supset E
5	C		2, 4 \supset E
6	A \supset C		3–5 \supset I

c. Derive: \sim B

1	A \equiv B		Assumption
2	\sim A		Assumption
3	B		A / \sim I
4	A		1, 3 \equiv E
5	\sim A		2 R
6	\sim B		3–5 \sim I

e. Derive: $A \supset [B \supset (C \supset D)]$

1	D	
2	A	Assumption
3	B	A / \supset I
4	C	A / \supset I
5	D	A / \supset I
6	$C \supset D$	1 R
7	$B \supset (C \supset D)$	4-5 \supset I
8	$A \supset [B \supset (C \supset D)]$	3-6 \supset I
		2-7 \supset I

g. Derive: $A \supset (D \supset C)$

1	$A \supset (B \supset C)$	
2	$D \supset B$	Assumption
3	A	Assumption
4	D	A / \supset I
5	$B \supset C$	A / \supset I
6	B	1, 3 \supset E
7	C	2, 4 \supset E
8	$D \supset C$	5, 6 \supset E
9	$A \supset (D \supset C)$	4-7 \supset I
		3-8 \supset I

i. Derive: $A \supset C$

1	$\sim A \vee B$	
2	$B \supset C$	Assumption
3	A	Assumption
4	$\sim A$	A / \supset I
5	$\sim C$	A / \vee E
6	A	A / \sim E
7	$\sim A$	3 R
8	C	4 R
9	B	5-7 \sim E
10	C	A / \vee E
11	C	2, 9 \supset E
12	$A \supset C$	1, 4-8, 9-10 \vee E
		3-11 \supset I

k. Derive: B

1	A \supset (C \supset B)		Assumption
2	\sim C \supset \sim A		Assumption
3	A		Assumption
4	\sim B		A / \sim E
5	C \supset B		1, 3 \supset E
6	C		A / \sim I
7	B		5, 6 \supset E
8	\sim B		4 R
9	\sim C		6-8 \sim I
10	\sim A		2, 9 \supset E
11	A		3 R
12	B		4-11 \sim E

*m. Derive: F & G

1	F \equiv G		Assumption
2	F \vee G		Assumption
3	F		A / \vee E
4	F		3 R
5	G		A / \vee E
6	F		1, 5 \equiv E
7	F		2, 3-4, 5-6 \vee E
8	G		1, 7 \equiv E
9	F & G		7, 8 &I

3. Theorems

a. Derive: A \supset (A \vee B)

1	A		A / \supset I
2	A \vee B		1 \vee I
3	A \supset (A \vee B)		1-2 \supset I

c. Derive: A \supset [B \supset (A & B)]

1	A		A / \supset I
2	B		A / \supset I
3	A & B		1, 2 &I
4	B \supset (A & B)		2-3 \supset I
5	A \supset [B \supset (A & B)]		1-4 \supset I

e. Derive: $(A \equiv B) \supset (A \supset B)$

1		A \equiv B	A / \supset I
2		A	A / \supset I
3		B	1, 2 \equiv E
4		A \supset B	2-3 \supset I
5		(A \equiv B) \supset (A \supset B)	1-4 \supset I

g. Derive: $(A \supset B) \supset [(C \supset A) \supset (C \supset B)]$

1		A \supset B	A / \supset I
2		C \supset A	A / \supset I
3		C	A / \supset I
4		A	2, 3 \supset E
5		B	1, 4 \supset E
6		C \supset B	3-5 \supset I
7		(C \supset A) \supset (C \supset B)	2-6 \supset I
8		(A \supset B) \supset [(C \supset A) \supset (C \supset B)]	1-7 \supset I

i. Derive: $[(A \supset B) \ \& \ \sim B] \supset \sim A$

1		(A \supset B) $\ \& \ \sim$ B	A / \supset I
2		A	A / \supset I
3		A \supset B	1 $\ \& \$ E
4		B	2, 3 \supset I
5		\sim B	1 $\ \& \$ E
6		\sim A	2-5 \sim I
7		[(A \supset B) $\ \& \ \sim$ B] $\supset \sim$ A	1-6 \supset I

k. Derive: $A \supset [B \supset (A \supset B)]$

1		A	A / \supset I
2		B	A / \supset I
3		A	A / \supset I
4		B	2 R
5		A \supset B	3-4 \supset I
6		B \supset (A \supset B)	2-5 \supset I
7		A \supset [B \supset (A \supset B)]	1-6 \supset I

m. Derive: $(A \supset B) \supset [\sim B \supset \sim (A \& D)]$

1	A \supset B		
2	~ B	A & D	A / \supset I
3	A	B	A / \supset I
4	~ B	~ (A & D)	A / \sim I
5	~ B \supset ~ (A & D)		3 &E
6	(A \supset B) \supset [~ B \supset ~ (A & D)]		1, 4 \supset E
7			2 R
8			3-6 \sim I
9			2-7 \supset I
			1-8 \supset I

4. Equivalence

a. Derive: $A \& \sim A$

1	B & ~ B		
2	~ (A & ~ A)	B	Assumption
3	~ B	~ A	A / ~ E
4	A & ~ A		1 &E
5			1 &E
			2-4 ~ E

Derive: $B \& \sim B$

1	A & ~ A		
2	~ (B & ~ B)	A	Assumption
3	~ A	~ A	A / ~ E
4	B & ~ B		1 &E
5			1 &E
			2-4 ~ E

c. Derive: $(A \vee B) \supset A$

1	B \supset A		
2	A \vee B	A	Assumption
3	A	A	A / \vee E
4	B	A	3 R
5	A		A / \vee E
6	A		1, 5 \supset E
7	(A \vee B) \supset A		2, 3-4, 5-6 \vee E
8			2-7 \supset I

Derive: $B \supset A$

1	(A \vee B) \supset A		Assumption
2	B		A / \supset I
3	A \vee B		2 \vee I
4	A		1, 3 \supset E
5	B \supset A		2-4 \supset I

e. Derive: $\sim (A \equiv B)$

1	(A & \sim B) \vee (B & \sim A)		Assumption
2	A & \sim B		A / \vee E
3	A \equiv B		A / \sim I
4	A		2 &E
5	B		3, 4 \equiv E
6	\sim B		2 &E
7	\sim (A \equiv B)		2-6 \sim I
8	B & \sim A		A / \vee E
9	A \equiv B		A / \sim I
10	B		8 &E
11	A		9, 10 \equiv E
12	\sim A		8 &E
13	\sim (A \equiv B)		9-12 \sim I
14	\sim (A \equiv B)		1, 2-6, 7-13 \vee E

Derive: (A & \sim B) \vee (B & \sim A)

1	\sim (A \equiv B)		Assumption
2	\sim [(A & \sim B) \vee (B & \sim A)]		A / \sim I
3	A		A / \equiv I
4	\sim B		A / \sim E
5	A & \sim B		3, 4 &I
6	(A & \sim B) \vee (B & \sim A)		5 \vee I
7	\sim [(A & \sim B) \vee (B & \sim A)]		2 R
8	B		4-7 \sim I
9	B		A / \supset I
10	\sim A		A / \sim E
11	B & \sim A		9, 10 &I
12	(A & \sim B) \vee (B & \sim A)		11 \vee I
13	\sim [(A & \sim B) \vee (B & \sim A)]		2 R
14	A		10-13 \sim E
15	A \equiv B		3-8, 9-14 \equiv I
14	\sim (A \equiv B)		1 R
15	(A & \sim B) \vee (B & \sim A)		2-14 \sim E

5. Inconsistency

a. Derive: $A \supset A, \sim (A \supset A)$

1	~ (A \supset A)	Assumption
2	A	A / \supset I
3	A	2 R
4	A \supset A	2-3 \supset I
5	~ (A \supset A)	1 R

c. Derive: $A, \sim A$

1	A \equiv B	Assumption
2	B \supset ~ A	Assumption
3	A	Assumption
4	A	3 R
5	B	1, 4 \equiv E
6	~ A	2, 5 \supset E

e. Derive: $A, \sim A$

1	A \supset ~ A	Assumption
2	~ A \supset A	Assumption
3	A	A / ~ I
4	~ A	1, 3 \supset E
5	A	3 R
6	~ A	A / ~ I
7	A	2, 6 \supset E

g. Derive: $A \vee B, \sim (A \vee B)$

1	~ (A \vee B)	Assumption
2	C \supset A	Assumption
3	~ C \supset A	Assumption
4	C	A / ~ I
5	A	2, 4 \supset E
6	A \vee B	5 \vee I
7	~ (A \vee B)	1 R
8	~ C	4-7 ~ I
9	B	3,8 \supset E
10	A \vee B	9 \vee I
11	~ (A \vee B)	1 R

i. Derive: $F \vee G, \sim (F \vee G)$

1	$\sim (F \vee G) \equiv (A \supset A)$		
2	$H \supset F$		Assumption
3	$\sim H \supset F$		Assumption
4	A		A / \supset I
5	A		4 R
6	A \supset A		4-5 \supset I
7	$\sim (F \vee G)$		1, 6 \equiv E
8	H		A / \sim I
9	F		2, 8 \supset E
10	F \vee G		9 \vee I
11	$\sim (F \vee G)$		7 R
12	$\sim H$		8-11 \sim I
13	F		3, 12 \supset E
14	F \vee G		13 \vee I

6. Derivability

a. Derive: $A \equiv B$

1	A \supset B		
2	$\sim A \supset \sim B$		Assumption
3	A		A / \equiv I
4	B		1, 3 \supset E
5	B		A / \equiv I
6	$\sim A$		A / \sim E
7	$\sim B$		2, 6 \supset E
8	B		5 R
9	A		6-8 \sim E
10	A \equiv B		3-4, 5-9 \equiv I

c. Derive: A

1	A $\equiv (\sim B \vee C)$		
2	B \supset C		Assumption
3	$\sim A$		A / \sim E
4	B		A / \sim I
5	C		2, 4 \supset E
6	$\sim B \vee C$		5 \vee I
7	A		1, 6 \equiv E
8	$\sim A$		3 R
9	$\sim B$		4-8 \sim I
10	$\sim B \vee C$		9 \vee I
11	A		1, 10 \equiv E
12	$\sim A$		3 R
13	A		3-12 \sim E

e. Derive: $B \vee D$

1	$B \vee (C \vee D)$	Assumption
2	$C \supset A$	Assumption
3	$A \supset \sim C$	Assumption
4	B	$A / \vee E$
5	$B \vee D$	$4 \vee I$
6	$C \vee D$	$A / \vee E$
7	C	$A / \vee E$
8	$\sim (B \vee D)$	$A / \sim E$
9	A	$2, 7 \supset E$
10	$\sim C$	$3, 9 \supset E$
11	C	$7 R$
12	$B \vee D$	$8-11 \sim E$
13	D	$A / \vee E$
14	$B \vee D$	$13 \vee I$
15	$B \vee D$	$6, 7-12, 13-14 \vee E$
16	$B \vee D$	$1, 4-5, 6-15 \vee E$

g. Derive: $(A \vee B) \supset \sim C$

1	$A \supset (D \& B)$	Assumption
2	$(\sim D \equiv B) \& (C \supset A)$	Assumption
3	$A \vee B$	$A / \supset I$
4	A	$A / \vee E$
5	C	$A / \sim I$
6	$D \& B$	$1, 4 \supset E$
7	$\sim D \equiv B$	$2 \& E$
8	B	$6 \& E$
9	$\sim D$	$7, 8 \equiv E$
10	D	$6 \& E$
11	$\sim C$	$5-10 \sim I$
12	B	$A / \vee E$
13	C	$A / \sim I$
14	$C \supset A$	$2 \& E$
15	A	$13, 14 \supset E$
16	$D \& B$	$1, 15 \supset E$
17	D	$16 \& E$
18	$\sim D \equiv B$	$2 \& E$
19	B	$16 \& E$
20	$\sim D$	$18, 19 \equiv E$
21	$\sim C$	$13-20 \sim I$
22	$\sim C$	$3, 4-11, 12-21 \vee E$
23	$(A \vee B) \supset \sim C$	$2-22 \supset I$

7. Validity

a. Derive: $\sim (C \equiv \sim A)$

1	$\sim (C \vee A)$	Assumption
2	$C \equiv \sim A$	$A / \sim I$
3	$\sim A$	$A / \sim E$
4	C	$2, 3 \equiv E$
5	$C \vee A$	$4 \vee I$
6	$\sim (C \vee A)$	$1 R$
7	A	$3-6 \sim E$
8	$C \vee A$	$7 \vee I$
9	$\sim (C \vee A)$	$1 R$
10	$\sim (C \equiv \sim A)$	$2-9 \sim I$

c. Derive: $A \equiv B$

1	$\sim A \ \& \ \sim B$	Assumption
2	A	$A / \equiv I$
3	$\sim B$	$A / \sim E$
4	$\sim A$	$1 \ \&E$
5	A	$2 R$
6	B	$3-5 \sim E$
7	B	$A / \equiv I$
8	$\sim A$	$A / \sim E$
9	$\sim B$	$1 \ \&E$
10	B	$7 R$
11	A	$8-10 \sim E$
12	$A \equiv B$	$2-6, 7-11 \equiv I$

e. Derive: $\sim H$

1	H $\equiv \sim (I \& \sim J)$		Assumption
2	$\sim I \equiv \sim H$		Assumption
3	J $\supset \sim I$		Assumption
4	H		A / $\sim I$
5	$\sim (I \& \sim J)$		1, 4 $\equiv E$
6	$\sim I$		A / $\sim E$
7	$\sim H$		2, 6 $\equiv E$
8	H		4 R
9	I		6-8 $\sim E$
10	J		A / $\sim I$
11	$\sim I$		3, 10 $\supset E$
12	I		9 R
13	$\sim J$		10-13 $\sim I$
14	I & $\sim J$		9, 13 & I
15	$\sim H$		4-14 $\sim I$

g. Derive: $H \vee \sim I$

1	(F \vee G) \vee (H $\vee \sim I$)		Assumption
2	F \supset H		Assumption
3	I $\supset \sim G$		Assumption
4	F \vee G		A / $\vee E$
5	F		A / $\vee E$
6	H		2, 5 $\supset E$
7	H $\vee \sim I$		6 $\vee I$
8	G		A / $\vee E$
9	I		A / $\sim I$
10	$\sim G$		3, 9 $\supset E$
11	G		8 R
12	$\sim I$		9-11 $\sim I$
13	H $\vee \sim I$		12 $\vee I$
14	H $\vee \sim I$		4, 5-7, 8-13 $\vee E$
15	H $\vee \sim I$		A / $\vee E$
16	H $\vee \sim I$		15 R
17	H $\vee \sim I$		1, 4-14, 15-16 $\vee E$

i. Derive: $F \vee (I \& \sim G)$

1	$\sim (F \vee \sim G) \equiv \sim (H \vee I)$		Assumption
2	$F \vee I$		Assumption
3	F		A / \vee E
5	$F \vee (I \& \sim G)$		3 \vee I
6	I		A / \vee E
7	$\sim (F \vee \sim G)$		A / \sim E
8	$\sim (H \vee I)$		1, 7 \equiv E
9	$H \vee I$		6 \vee I
10	$F \vee \sim G$		7-9 \sim E
11	F		A / \vee E
12	$F \vee (I \& \sim G)$		11 \vee I
13	$\sim G$		A / \vee E
14	$I \& \sim G$		6, 13 $\&$ I
15	$F \vee (I \& \sim G)$		15 \vee I
16	$F \vee (I \& \sim G)$		10, 11-12, 13-15 \vee E
17	$F \vee (I \& \sim G)$		2, 3-5, 6-16 \vee E

k. Derive: $(\sim A \equiv \sim C) \supset (\sim A \equiv D)$

1	$(\sim A \equiv \sim C) \equiv (B \equiv \sim D)$		Assumption
2	$\sim A \supset \sim B$		Assumption
3	$C \supset \sim D$		Assumption
4	$\sim A \equiv \sim C$		A / \supset I
5	$\sim A$		A / \equiv I
6	$\sim D$		A / \sim E
7	$B \equiv \sim D$		1, 4 \supset E
8	B		6, 7 \equiv E
9	$\sim B$		2, 5 \supset E
10	D		6-9 \sim E
11	D		A / \equiv I
12	C		A / \sim I
13	$\sim D$		3, 12 \supset E
14	D		11 R
15	$\sim C$		12-14 \sim I
16	$\sim A$		4, 15 \equiv E
17	$\sim A \equiv D$		5-10, 11-16 \equiv I
20	$(\sim A \equiv \sim C) \supset (\sim A \equiv D)$		4-17 \supset I

m. Derive: $\sim E$

1	$\sim (A \supset B) \ \& \ (C \ \& \ \sim D)$	Assumption
2	$(B \vee \sim A) \vee [(C \ \& \ E) \supset D]$	Assumption
3	E	$A / \sim I$
4	B $\vee \sim A$	$A / \vee E$
5	B	$A / \vee E$
6	A	$A / \supset I$
7	B	5 R
8	A $\supset B$	6-7 $\supset I$
9	$\sim A$	$A / \vee E$
10	A	$A / \supset I$
11	$\sim B$	$A / \sim E$
12	A	10 R
13	$\sim A$	9 R
14	B	11-13 $\sim E$
15	A $\supset B$	10-14 $\supset I$
16	A $\supset B$	4, 5-8, 9-15 $\vee E$
17	$(C \ \& \ E) \supset D$	$A / \vee E$
18	$\sim (A \supset B)$	$A / \sim E$
19	C $\ \& \ \sim D$	1 $\ \& \ E$
20	$\sim D$	19 $\ \& \ E$
21	C	19 $\ \& \ E$
22	C $\ \& \ E$	3, 21 $\ \& \ I$
22	D	17, 22 $\supset E$
23	A $\supset B$	18-22 $\sim E$
24	A $\supset B$	2, 4-16, 17-23 $\vee E$
25	$\sim (A \supset B)$	1 $\ \& \ E$
26	$\sim E$	3-25 $\sim I$

9. Theorems

a. Derive: $\sim (A \supset B) \supset \sim (A \equiv B)$

1	$\sim (A \supset B)$	$A / \supset I$
2	A $\equiv B$	$A / \sim I$
3	A	$A / \supset I$
4	B	2, 3 $\equiv E$
5	A $\supset B$	3-4 $\supset I$
6	$\sim (A \supset B)$	1 R
7	$\sim (A \equiv B)$	2-6 $\sim I$
8	$\sim (A \supset B) \supset \sim (A \equiv B)$	1-7 $\supset I$

c. Derive: $(A \supset B) \vee (B \supset A)$

1	~ $[(A \supset B) \vee (B \supset A)]$		A / ~ E
2	B		A / ~ I
3	A		A / \supset I
4	B		2 R
5	$A \supset B$		3-4 \supset I
6	$(A \supset B) \vee (B \supset A)$		5 \vee I
7	~ $[(A \supset B) \vee (B \supset A)]$		1 R
8	~ B		2-7 \vee I
9	B		A / \supset I
10	~ A		A / ~ E
11	B		9 R
12	~ B		8 R
13	A		10-12 ~ E
14	$B \supset A$		9-13 \supset I
15	$(A \supset B) \vee (B \supset A)$		14 \vee I
16	~ $[(A \supset B) \vee (B \supset A)]$		1 R
17	$(A \supset B) \vee (B \supset A)$		1-16 ~ E

e. Derive: $[(A \vee B) \supset C] \equiv [(A \supset C) \& (B \supset C)]$

1	$(A \vee B) \supset C$		A / \equiv I
2	A		A / \supset I
3	$A \vee B$		2 \vee I
4	C		1, 3 \supset E
5	$A \supset C$		2-4 \supset I
6	B		A / \supset I
7	$A \vee B$		6 \vee I
8	C		1, 7 \supset E
9	$B \supset C$		6-8 \supset I
10	$(A \supset C) \& (B \supset C)$		5, 9 $\&$ I
11	$(A \supset C) \& (B \supset C)$		A / \equiv I
12	$A \vee B$		A / \supset I
13	A		A / \vee E
14	$A \supset C$		11 $\&$ E
15	C		13, 14 \supset E
16	B		A / \vee E
17	$B \supset C$		11 $\&$ E
18	C		16, 17 \supset E
19	C		12, 13-15, 16-18 \vee E
20	$(A \vee B) \supset C$		12-19 \supset I
21	$[(A \vee B) \supset C] \equiv [(A \supset C) \& (B \supset C)]$		1-10, 11-20 \equiv I

g. Derive: $\sim (A \equiv B) \equiv (A \equiv \sim B)$

1	$\sim (A \equiv B)$	A / \equiv I
2	A	A / \equiv I
3	B	A / \sim I
4	A	A / \equiv I
5	B	3 R
6	B	A / \equiv I
7	A	2 R
8	A \equiv B	4-5, 6-7 \equiv I
9	$\sim (A \equiv B)$	1 R
10	$\sim B$	3-9 \sim I
11	$\sim B$	A / \equiv I
12	$\sim A$	A / \sim E
13	A	A / \equiv I
14	$\sim B$	A / \sim E
15	A	13 R
16	$\sim A$	12 R
17	B	14-16 \sim E
18	B	A / \equiv I
19	$\sim A$	A / \sim E
20	B	18 R
21	$\sim B$	11 R
22	A	19-21 \sim E
23	A \equiv B	13-17, 18-22
24	$\sim (A \equiv B)$	1 R
25	A	12-24 \sim E
26	A $\equiv \sim B$	2-10, 11-25 \equiv I
27	A $\equiv \sim B$	A / \equiv I
28	A \equiv B	A / \sim I
29	B	A / \sim I
30	A	28, 29 \equiv E
31	$\sim B$	27, 30 \equiv E
32	B	29 R
33	$\sim B$	29-32 \sim I
34	$\sim B$	A / \sim E
35	A	27, 34 \equiv E
36	B	28, 35 \equiv E
37	$\sim B$	34 R
38	B	34-37 \sim E
39	$\sim (A \equiv B)$	28-38 \sim I
40	$\sim (A \equiv B)$	

10. Equivalence

a. Derive: $\sim \sim A$

1	A	Assumption
2	$\sim A$	A / \sim I
3	A	1 R
4	$\sim A$	2 R
5	$\sim \sim A$	2-4 \sim I

Derive: A

1	$\sim \sim A$	Assumption
2	$\sim A$	A / \sim E
3	$\sim A$	2 R
4	$\sim \sim A$	1 R
5	A	2-4 \sim E

c. Derive: $A \vee A$

1	A	Assumption
2	$A \vee A$	A / \vee I

Derive: $A \vee A$

1	$A \vee A$	Assumption
2	A	A / \vee E
3	A	2 R
4	A	1, 2-3, 2-3 \vee E

e. Derive: $B \vee A$

1	$A \vee B$	Assumption
2	A	A / \vee E
3	$B \vee A$	2 \vee I
4	B	A / \vee E
5	$B \vee A$	4 \vee I
6	$B \vee A$	1, 2-3, 4-5 \vee E

Derive: $A \vee B$

1	$B \vee A$	Assumption
2	B	$A / \vee E$
3	$A \vee B$	$2 \vee I$
4	A	$A / \vee E$
5	$A \vee B$	$4 \vee I$
6	$A \vee B$	$1, 2-3, 4-5 \vee E$

g. Derive: $(A \vee B) \vee C$

1	$A \vee (B \vee C)$	Assumption
2	A	$A / \vee E$
3	$A \vee B$	$2 \vee I$
4	$(A \vee B) \vee C$	$3 \vee I$
5	$B \vee C$	$A / \vee E$
6	B	$A / \vee E$
7	$A \vee B$	$6 \vee I$
9	$(A \vee B) \vee C$	$7 \vee I$
10	C	$A / \vee I$
11	$(A \vee B) \vee C$	$10 \vee I$
12	$(A \vee B) \vee C$	$5, 6-9, 10-11 \vee I$
13	$(A \vee B) \vee C$	$1, 2-4, 5-12 \vee E$

Derive: $A \vee (B \vee C)$

1	$(A \vee B) \vee C$	Assumption
2	$A \vee B$	$A / \vee E$
3	A	$A / \vee E$
4	$A \vee (B \vee C)$	$3 \vee I$
5	B	$A / \vee E$
6	$B \vee C$	$5 \vee I$
7	$A \vee (B \vee C)$	$6 \vee I$
8	$A \vee (B \vee C)$	$2, 3-4, 5-7 \vee E$
9	C	$A / \vee E$
10	$B \vee C$	$9 \vee I$
11	$A \vee (B \vee C)$	$10 \vee I$
12	$A \vee (B \vee C)$	$1, 2-8, 9-11 \vee E$

i. Derive: $\sim B \supset \sim A$

1	A \supset B	Assumption
2	<div style="border-left: 1px solid black; padding-left: 5px;"> \sim B </div>	A / \supset I
3	<div style="border-left: 1px solid black; padding-left: 5px;"> <div style="border-left: 1px solid black; padding-left: 5px;">A</div> </div>	A / \sim E
4	<div style="border-left: 1px solid black; padding-left: 5px;"> <div style="border-left: 1px solid black; padding-left: 5px;">B</div> </div>	1, 3 \supset E
5	<div style="border-left: 1px solid black; padding-left: 5px;"> <div style="border-left: 1px solid black; padding-left: 5px;">\sim B</div> </div>	2 R
6	<div style="border-left: 1px solid black; padding-left: 5px;">\sim A</div>	3-5 \sim I
7	\sim B \supset \sim A	2-6 \supset I

Derive: A \supset B

1	\sim B \supset \sim A	Assumption
2	<div style="border-left: 1px solid black; padding-left: 5px;">A</div>	A / \supset I
3	<div style="border-left: 1px solid black; padding-left: 5px;"> <div style="border-left: 1px solid black; padding-left: 5px;">\sim B</div> </div>	A / \sim E
4	<div style="border-left: 1px solid black; padding-left: 5px;"> <div style="border-left: 1px solid black; padding-left: 5px;">A</div> </div>	2 R
5	<div style="border-left: 1px solid black; padding-left: 5px;">\sim A</div>	1, 3 \supset E
6	<div style="border-left: 1px solid black; padding-left: 5px;">B</div>	3-5 \sim E
7	A \supset B	2-6 \supset I

k. Derive: A \equiv B

1	(A & B) \vee (\sim A & \sim B)	Assumption
2	<div style="border-left: 1px solid black; padding-left: 5px;">A & B</div>	A / \vee E
3	<div style="border-left: 1px solid black; padding-left: 5px;"> <div style="border-left: 1px solid black; padding-left: 5px;">A</div> </div>	A / \equiv I
4	<div style="border-left: 1px solid black; padding-left: 5px;"> <div style="border-left: 1px solid black; padding-left: 5px;">B</div> </div>	2 &E
5	<div style="border-left: 1px solid black; padding-left: 5px;"> <div style="border-left: 1px solid black; padding-left: 5px;">B</div> </div>	A / \equiv I
6	<div style="border-left: 1px solid black; padding-left: 5px;">A</div>	2 &E
7	A \equiv B	3-4, 5-6 \equiv I
8	\sim A & \sim B	A / \vee E
9	<div style="border-left: 1px solid black; padding-left: 5px;">A</div>	A / \equiv I
10	<div style="border-left: 1px solid black; padding-left: 5px;"> <div style="border-left: 1px solid black; padding-left: 5px;">\sim B</div> </div>	A / \sim E
11	<div style="border-left: 1px solid black; padding-left: 5px;"> <div style="border-left: 1px solid black; padding-left: 5px;">A</div> </div>	9 R
12	<div style="border-left: 1px solid black; padding-left: 5px;"> <div style="border-left: 1px solid black; padding-left: 5px;">\sim A</div> </div>	8 &E
13	<div style="border-left: 1px solid black; padding-left: 5px;">B</div>	10-12 \sim E
14	<div style="border-left: 1px solid black; padding-left: 5px;">B</div>	A / \equiv I
15	<div style="border-left: 1px solid black; padding-left: 5px;"> <div style="border-left: 1px solid black; padding-left: 5px;">\sim A</div> </div>	A / \sim E
16	<div style="border-left: 1px solid black; padding-left: 5px;"> <div style="border-left: 1px solid black; padding-left: 5px;">B</div> </div>	14 R
17	<div style="border-left: 1px solid black; padding-left: 5px;"> <div style="border-left: 1px solid black; padding-left: 5px;">\sim B</div> </div>	8 &E
18	<div style="border-left: 1px solid black; padding-left: 5px;">A</div>	15-17 \sim E
19	A \equiv B	9-13, 14-18 \equiv I
20	A \equiv B	1, 2-7, 8-19 \vee E

Derive: $(A \& B) \vee (\sim A \& \sim B)$

1	A \equiv B	Assumption
2	$\sim [(A \& B) \vee (\sim A \& \sim B)]$	A / \sim E
3	A	A / \sim I
4	B	1, 3 \equiv E
5	A & B	3, 4 &I
6	$(A \& B) \vee (\sim A \& \sim B)$	5 \vee I
7	$\sim [(A \& B) \vee (\sim A \& \sim B)]$	2 R
8	\sim A	3-7 \sim I
9	B	A / \sim I
10	A	1, 9 \equiv E
11	\sim A	8 R
12	\sim B	9-11 \sim I
13	\sim A & \sim B	8, 12 &I
14	$(A \& B) \vee (\sim A \& \sim B)$	13 \vee I
15	$\sim [(A \& B) \vee (\sim A \& \sim B)]$	2 R
16	$(A \& B) \vee (\sim A \& \sim B)$	2-15 \sim E

m. Derive: $(A \vee B) \& (A \vee C)$

1	A \vee (B & C)	Assumption
2	A	A / \vee E
3	A \vee B	2 \vee I
4	A \vee C	2 \vee I
5	$(A \vee B) \& (A \vee C)$	3, 4 &I
6	B & C	A / \vee E
7	B	6 & E
8	A \vee B	7 \vee I
9	C	6 &E
10	A \vee C	9 \vee I
11	$(A \vee B) \& (A \vee C)$	8, 10 &I
12	$(A \vee B) \& (A \vee C)$	1, 2-5, 6-11 \vee E

Derive: $A \vee (B \& C)$

1	(A \vee B) & (A \vee C)		Assumption
2	A \vee B		1 &E
3	A		A / \vee E
4	A \vee (B & C)		3 \vee I
5	B		A / \vee E
6	A \vee C		1 &E
7	A		A / \vee E
8	A \vee (B & C)		7 \vee I
9	C		A / \vee E
10	B & C		5, 9 &I
11	A \vee (B & C)		10 \vee I
12	A \vee (B & C)		6, 7-8, 9-11 \vee E
13	A \vee (B & C)		2, 3-4, 5-12 \vee E

o. Derive: $\sim A \vee \sim B$

1	$\sim (A \& B)$		Assumption
2	$\sim (\sim A \vee \sim B)$		A / \sim E
3	$\sim A$		A / \sim E
4	$\sim A \vee \sim B$		3 \vee I
5	$\sim (\sim A \vee \sim B)$		2 R
6	A		3-5 \sim E
7	$\sim B$		A / \sim E
8	$\sim A \vee \sim B$		7 \vee I
9	$\sim (\sim A \vee \sim B)$		2 R
10	B		7-9 \sim E
11	A & B		6, 10 &I
12	$\sim (A \& B)$		1 R
13	$\sim A \vee \sim B$		2-12 \sim E

Derive: $\sim (A \& B)$

1	$\sim A \vee \sim B$		
2	$A \& B$		Assumption
3	$\sim A$		$A / \sim I$
4	$\sim A$		$A / \vee E$
5	$\sim B$		3 R
6	$\sim B$		$A / \vee E$
7	A		$A / \sim I$
8	B		2 &E
9	$\sim B$		5 R
10	$\sim A$		6-8 $\sim I$
11	A		1, 3-4, 5-9 $\vee E$
12	$\sim (A \& B)$		2 &E
			2-11 $\sim I$

12. Inconsistency

a. Derive: B, $\sim B$

1	$(A \supset B) \& (A \supset \sim B)$		
2	$(C \supset A) \& (\sim C \supset A)$		Assumption
3	$A \supset B$		Assumption
4	$A \supset \sim B$		1 &E
5	C		1 &E
6	$C \supset A$		$A / \sim I$
7	A		2 &E
8	B		5, 6 $\supset E$
9	$\sim B$		3, 7 $\supset E$
10	$\sim C$		4, 7 $\supset E$
11	$\sim C \supset A$		5-9 $\sim I$
12	A		2 &E
13	B		10, 11 $\supset E$
14	$\sim B$		3, 12 $\supset E$
			4, 12 $\supset E$

c. Derive: $A, \sim A$

1	C $\equiv \vee A$		
2	C $\equiv A$		Assumption
3	A		A / \sim I
4	C		2, 3 \equiv E
5	$\sim A$		1, 4 \equiv E
6	A		3 R
7	$\sim A$		3-6 \sim I
8	$\sim A$		A / \sim E
9	C		1, 8 \equiv E
10	A		2, 9 \equiv E
11	$\sim A$		8 R
12	A		8-11 \sim E

e. Derive: $A, \sim A$

1	$\sim [(A \vee B) \vee C]$		
2	A $\equiv \sim C$		Assumption
3	A		A / \sim I
4	A $\vee B$		3 \vee I
5	(A $\vee B$) $\vee C$		4 \vee I
6	$\sim [(A \vee B) \vee C]$		1 R
7	$\sim A$		3-6 \sim I
8	$\sim A$		A / \sim E
9	C		A / \sim I
10	A		2, 9 \equiv E
11	$\sim A$		8 R
12	$\sim C$		9-11 \sim I
13	A		2, 12 \equiv E
14	$\sim A$		8 R
15	A		8-14 \sim E

g. Derive: $B, \sim B$

1	A & (B \vee C)		Assumption
2	$(\sim C \vee H) \& (H \supset \sim H)$		Assumption
3	$\sim B$		Assumption
4	B \vee C		1 &E
5	B		A / \vee E
6	B		5 R
7	C		A / \vee E
8	$\sim C \vee H$		2 &E
9	$\sim C$		A / \vee E
10	$\sim B$		A / \sim E
11	C		7 R
12	$\sim C$		9 R
13	B		10–12 \sim E
14	H		A / \vee E
15	$\sim B$		A / \sim E
16	H $\supset \sim H$		2 &E
17	$\sim H$		14, 16 \supset E
18	B		15, 17 \sim E
19	B		8, 9–13, 14–18 \vee E
20	B		4, 5–6, 7–19 \vee E
21	$\sim B$		3 R

13. Validity

a. Derive: M

1	S & F		Assumption
2	F \supset B		Assumption
3	$(B \& \sim M) \supset \sim S$		Assumption
4	$\sim M$		A / \sim E
5	F		1 &E
6	B		2, 5 \supset E
7	B & $\sim M$		6, 4 &I
8	$\sim S$		3, 7 \supset E
9	S		1 &E
10	M		4–9 \sim E

c. Derive: $\sim J$

1	(C \supset \sim R) & (R \supset L)		Assumption
2	C \equiv (C \vee L)		Assumption
3	J \supset R		Assumption
4	J		A / \sim I
5	R		3, 4 \supset E
6	R \supset L		1 &E
7	L		5, 6 \supset E
8	C \vee L		7 \vee I
9	C		2, 8 \equiv E
10	C \supset \sim R		1 &E
11	\sim R		9, 10 \supset E
12	\sim J		4-11 \sim I

e. Derive: $\sim M$

1	\sim (R \vee W)		Assumption
2	(R \equiv M) \vee [(M \vee G) \supset (W \equiv M)]		Assumption
3	M		A / \sim I
4	R \equiv M		A / \vee E
5	R		3, 4 \equiv E
6	R \vee W		5 \vee I
7	(M \vee G) \supset (W \equiv M)		A / \vee E
8	M \vee G		3 \vee I
9	W \equiv M		7, 8 \supset E
10	W		3, 9 \equiv E
11	R \vee W		10 \vee I
12	R \vee W		2, 4-6, 7-11 \vee E
13	\sim (R \vee W)		1 R
14	\sim M		3-13 \sim I

g. Derive: $H \supset J$

1	(H & T) \supset J	Assumption
2	(M \supset D) & (\sim D \supset M)	Assumption
3	\sim T \equiv (\sim D & M)	Assumption
4	H	A / \supset I
5	\sim J	A / \sim E
6	T	A / \sim I
7	H & T	4, 6 &I
8	J	1, 7 \supset E
9	\sim J	5 R
10	\sim T	6-9 \sim I
11	\sim D & M	3, 10 \equiv E
12	M \supset D	2 &E
13	M	11 &E
14	D	12, 13 \supset E
15	\sim D	11 &E
16	J	5-15 \sim E
17	H \supset J	4-16 \supset I

i. Derive: $L \supset T$

1	$L \supset (C \vee T)$	Assumption
2	$(\sim L \vee B) \ \& \ (\sim B \vee \sim C)$	Assumption
3	L	$A / \supset I$
4	$C \vee T$	$1, 3 \supset E$
5	C	$A / \vee E$
6	$\sim B \vee \sim C$	$2 \ \& E$
7	$\sim B$	$A / \vee E$
8	$\sim L \vee B$	$2 \ \& E$
9	$\sim L$	$A / \vee E$
10	$\sim T$	$A / \sim E$
11	L	$3 \ R$
12	$\sim L$	$9 \ R$
13	T	$10\text{--}12 \ \sim E$
14	B	$A / \vee E$
15	$\sim T$	$A / \sim E$
16	B	$14 \ R$
17	$\sim B$	$7 \ R$
18	T	$15\text{--}17 \ \sim E$
19	T	$8, 9\text{--}13, 14\text{--}18 \ \vee E$
20	$\sim C$	$A / \vee E$
21	$\sim T$	$A / \sim E$
22	$\sim C$	$20 \ R$
23	C	$5 \ R$
24	T	$21\text{--}23 \ \sim E$
25	T	$6, 7\text{--}19, 20\text{--}24 \ \vee E$
26	T	$A / \vee E$
27	T	$26 \ R$
28	T	$4, 5\text{--}25, 26\text{--}27 \ \vee E$
29	$L \supset T$	$3\text{--}28 \supset I$

14. Inconsistency

a.	1	(M \supset B) & (B \supset P)		Assumption
	2	M & \sim P		Assumption
	3	M		2 &E
	4	M \supset B		1 &E
	5	B		3, 4 \supset E
	6	B \supset P		1 &E
	7	P		5, 6 \supset E
	8	\sim P		2 &E
	c.	B \supset I		Assumption
	2	(\sim B & \sim I) \supset C		Assumption
	3	\sim C & \sim I		Assumption
	4	B		A / \sim I
	5	I		1, 4 \supset E
	6	\sim I		3 &E
	7	\sim B		4–6 \sim I
	8	\sim I		3 &E
	9	\sim B & \sim I		7, 8 &I
	10	C		2, 9 \supset E
	11	\sim C		3 &E
	e.	M \vee (F \supset T)		Assumption
	2	N \equiv \sim T		Assumption
	3	(F & N) & \sim M		Assumption
	4	M		A / \vee E
	5	M		4 R
	6	F \supset T		A / \vee E
	7	\sim M		A / \sim E
	8	F & N		3 &E
	9	F		8 &E
	10	T		6, 9 \supset E
	11	N		8 &E
	12	\sim T		2, 11 \equiv E
	13	M		7–12 \sim E
	14	M		1, 4–5, 6–13 \vee E
	15	\sim M		3 &E

15.a. We do not want this rule as a rule of *SD* because it is not truth-preserving. The truth of $\mathbf{P} \vee \mathbf{Q}$ does not entail the truth of \mathbf{P} .

c. We can show that Reiteration is dispensable by explaining how do derive **P** whenever **P** occurs on an earlier accessible line, without using Reiteration. Assume that **P** occurs on an accessible line **i** and that we want to derive **P** on a later line. We can do this as follows:

i	P	...
n	P & P	i, i & I
n + 1	P	n & E

e. Assume that **P** is a theorem is *SD*. Now consider any argument has $\sim P$ as one of its premises:

Q₁
Q₂
...
Q_n
<u>~ P</u>
R

We can derive **R** from the set consisting of the premises as follows:

1	Q₁	Assumption
2	Q₂	Assumption
	...	
n	Q_n	Assumption
n + 1	<u>~ P</u>	Assumption
n + 2	<u>~ R</u>	A / ~ E
n + 3	<u>~ P</u>	
n + 4		n + 1 R
	...	
n + k	P	
n + k + 1	R	n + 2 - n + k ~ E

Here lines **n + 4** through **n + k** consists of the derivation of **P** from no primary assumptions. We know there is such a derivation because we know **P** is a theorem of *SD*.

e. If **P** is a theorem is *SD* then any argument of *SL* that has $\sim P$ among its premises is valid in *SD*. We can construct a derivation of the conclusion, call it **Q** by taking the premises of the argument as our primary assumptions.

$\sim \mathbf{P}$ will be one of these assumptions. Next assume $\sim \mathbf{Q}$, derive both \mathbf{P} and $\sim \mathbf{P}$, and obtain \mathbf{Q} by Negation Elimination. We can obtain $\sim \mathbf{P}$ by Reiteration since it is one of the primary assumptions of the derivation. We can obtain \mathbf{P} because it is a theorem of SD and therefore can be derived from the empty set. If it can be derived from the empty set it can also be derived from the set consisting of the premises of the argument, by inserting the derivation of \mathbf{P} from the empty set within the scope of the assumption $\sim \mathbf{Q}$.

16. We here make use of a result established in Sections 6.3 and 6.4,

$\Gamma \vdash \mathbf{P}$ in SD if and only if $\Gamma \models \mathbf{P}$

a. Assume that a given argument is valid in SD . Then we know that its conclusion is derivable in SD from the set consisting of its premises. By the above result it follows that the conclusion of the argument is truth-functionally entailed by the set consisting of the premises of the argument. Therefore there is no truth-value assignment on which the members of the set, which are just the premises of the argument, are true and the conclusion of the argument false. So the argument is truth-functionally valid. Conversely, assume that the given argument is truth-functionally valid. So there is no truth-value assignment on which the premises of the argument are true and the conclusion false. From this it follows that the set consisting of the premises of the argument truth-functionally entails the conclusion of the argument. And by the above result it next follows that the conclusion of the argument is derivable from the set consisting of the premises of the argument, and from this it follows that the argument is valid in SD .

d. Assume that sentences \mathbf{P} and \mathbf{Q} of SL are equivalent in SD . Then each can be derived from the unit set of the other. By the above result it follows that the unit set of each truth-functionally entails the other. So there is no truth-value assignment on which \mathbf{P} is true and \mathbf{Q} false, and no truth-value assignment on which \mathbf{Q} is true and \mathbf{P} false. So \mathbf{P} and \mathbf{Q} are truth-functionally equivalent.

Exercises 5.4E

Derive: $\sim \mathbf{D}$

1	D \supset E	Assumption
2	E \supset (Z & W)	Assumption
3	$\sim Z \vee \sim W$	Assumption
4	$\sim (Z \& W)$	3 DeM
5	$\sim E$	2, 4 MT
6	$\sim D$	1, 5 MT

c. Derive: K

1	(W \supset S) & \sim M	Assumption
2	$(\sim$ W \supset H) \vee M	Assumption
3	$(\sim$ S \supset H) \supset K	Assumption
4	W \supset S	1 &E
5	\sim S \supset \sim W	4 Trans
6	\sim M	1 &E
7	\sim W \supset H	2, 6 DS
8	\sim S \supset H	5, 7 HS
9	K	3, 8 \supset E

e. Derive: C

1	(M \vee B) \vee (C \vee G)	Assumption
2	\sim B & (\sim G & \sim M)	Assumption
3	\sim B	2 &E
4	(B \vee M) \vee (C \vee G)	1 Com
5	B \vee [M \vee (C \vee G)]	4 Assoc
6	M \vee (C \vee G)	3, 5 DS
7	\sim G & \sim M	2 &E
8	\sim G	7 &E
9	(M \vee C) \vee G	6 Assoc
10	M \vee C	8, 9 DS
11	\sim M	7 &E
12	C	10, 11 DS

2. Validity

a. Derive: $Y \equiv Z$

1	\sim Y \supset \sim Z	Assumption										
2	\sim Z \supset \sim X	Assumption										
3	\sim X \supset \sim Y	Assumption										
4	<table style="border-collapse: collapse; margin-left: 5px;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">Y</td> <td style="padding-left: 5px;"></td> </tr> <tr> <td colspan="2" style="border-top: 1px solid black; padding-top: 5px;"></td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">\sim Z \supset \sim Y</td> <td style="padding-left: 5px;">2, 3 HS</td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">Y \supset Z</td> <td style="padding-left: 5px;">5 Trans</td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">Z</td> <td style="padding-left: 5px;">4, 6 \supsetE</td> </tr> </table>	Y				\sim Z \supset \sim Y	2, 3 HS	Y \supset Z	5 Trans	Z	4, 6 \supset E	A / \equiv E
Y												
\sim Z \supset \sim Y	2, 3 HS											
Y \supset Z	5 Trans											
Z	4, 6 \supset E											
8	<table style="border-collapse: collapse; margin-left: 5px;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">Z</td> <td style="padding-left: 5px;"></td> </tr> <tr> <td colspan="2" style="border-top: 1px solid black; padding-top: 5px;"></td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">Z \supset Y</td> <td style="padding-left: 5px;">1 Trans</td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">Y</td> <td style="padding-left: 5px;">8, 9 \supsetE</td> </tr> </table>	Z				Z \supset Y	1 Trans	Y	8, 9 \supset E	A / \equiv I		
Z												
Z \supset Y	1 Trans											
Y	8, 9 \supset E											
11	Y \equiv Z	4-7, 8-10 \equiv I										

c. Derive: $I \supset \sim D$

1		(F & G) \vee (H & \sim I)	Assumption
2		$I \supset \sim$ (F & D)	Assumption
3		I	A / \supset I
4		\sim (F & D)	2, 3 \supset E
5		\sim F \vee \sim D	4 DeM
6		$\sim \sim$ I	3 DN
7		\sim H \vee $\sim \sim$ I	6 \vee I
8		\sim (H & \sim I)	7 DeM
9		F & G	1, 8 DS
10		F	9 &E
11		$\sim \sim$ F	10 DN
12		\sim D	5, 11 DS
13		$I \supset \sim$ D	3–12 \supset I

e. Derive: $I \vee H$

1		F \supset (G \supset H)	Assumption
2		\sim I \supset (F \vee H)	Assumption
3		F \supset G	Assumption
4		\sim I	A / \supset I
5		F \vee H	2, 4 \supset E
6		\sim H	A / \sim E
7		F	5, 6 DS
8		G	3, 7 \supset E
9		G \supset H	1, 7 \supset E
10		\sim G	6, 9 MT
11		H	6–10 \sim E
12		\sim I \supset H	4–11 \supset I
13		$\sim \sim$ I \vee H	12 Impl
14		I \vee H	13 DN

g. Derive: $X \equiv Y$

1	$[(X \& Z) \& Y] \vee (\sim X \supset \sim Y)$	Assumption
2	$X \supset Z$	Assumption
3	$Z \supset Y$	Assumption
4	X	A / \equiv I
5	Z	2, 4 \supset E
6	Y	3, 5 \supset E
7	Y	A / \equiv I
8	$(X \& Z) \& Y$	A / \vee E
9	$X \& Z$	8 &E
10	X	9 &E
11	$\sim X \supset \sim Y$	A / \vee E
12	$Y \supset X$	11 Trans
13	X	7, 12 \supset E
14	X	1, 8–10, 11–13 \vee E
15	$X \equiv Y$	4–6, 7–14 \equiv I

3. Theorems

a. Derive: $A \vee \sim A$

1	$\sim (A \vee \sim A)$	A / \sim E
2	$\sim A \& \sim \sim A$	1 DeM
3	$\sim A$	2 &E
4	$\sim \sim A$	2 &E
5	$A \vee \sim A$	1–4 \sim E

c. Derive: $A \vee [(\sim A \vee B) \& (\sim A \vee C)]$

1	$\sim A$	A / \supset I
2	$\sim A \vee (B \& C)$	1 \vee I
3	$(\sim A \vee B) \& (\sim A \vee C)$	2 Dist
4	$\sim A \supset [(\sim A \vee B) \& (\sim A \vee C)]$	1–3 \supset I
5	$\sim \sim A \vee [(\sim A \vee B) \& (\sim A \vee C)]$	4 Impl
6	$A \vee [(\sim A \vee B) \& (\sim A \vee C)]$	5 DN

e. Derive: $[A \supset (B \& C)] \equiv [(\sim B \vee \sim C) \supset \sim A]$

1		$A \supset (B \& C)$	A / \equiv I
2		$\sim (B \& C) \supset \sim A$	1 Trans
3		$(\sim B \vee \sim C) \supset \sim A$	2 DeM
4		$(\sim B \vee \sim C) \supset \sim A$	A / \equiv I
5		$\sim (B \& C) \supset \sim A$	4 DeM
6		$A \supset (B \& C)$	5 Trans
7		$[A \supset (B \& C)] \equiv [(\sim B \vee \sim C) \supset \sim A]$	1-3, 4-6 \equiv I

g. Derive: $[A \supset (B \equiv C)] \equiv (A \supset [(\sim B \vee C) \& (\sim C \vee B)])$

1		$A \supset (B \equiv C)$	A / \equiv I
2		$A \supset [(B \supset C) \& (C \supset B)]$	1 Equiv
3		$A \supset [(\sim B \vee C) \& (C \supset B)]$	2 Impl
4		$A \supset [(\sim B \vee C) \& (\sim C \vee B)]$	3 Impl
5		$A \supset [(\sim B \vee C) \& (\sim C \vee B)]$	A / \equiv I
6		$A \supset [(B \supset C) \& (\sim C \vee B)]$	5 Impl
7		$A \supset [(B \supset C) \& (C \supset B)]$	6 Impl
8		$A \supset (B \equiv C)$	7 Equiv
9		$[A \supset (B \equiv C)] \equiv (A \supset [(\sim B \vee C) \& (\sim C \vee B)])$	1-4, 5-8 \equiv I

i. Derive: $[\sim A \supset (\sim B \supset C)] \supset [(A \vee B) \vee (\sim \sim B \vee C)]$

1		$\sim A \supset (\sim B \supset C)$	A / \supset I
2		$\sim \sim A \vee (\sim B \supset C)$	1 Impl
3		$\sim \sim A \vee (\sim \sim B \vee C)$	2 Impl
4		$A \vee (\sim \sim B \vee C)$	3 DN
5		$A \vee [(\sim \sim B \vee \sim \sim B) \vee C]$	4 Idem
6		$A \vee [\sim \sim B \vee (\sim \sim B \vee C)]$	5 Assoc
7		$(A \vee \sim \sim B) \vee (\sim \sim B \vee C)$	6 Assoc
8		$(A \vee B) \vee (\sim \sim B \vee C)$	7 DN
9		$[\sim A \supset (\sim B \supset C)] \supset [(A \vee B) \vee (\sim \sim B \vee C)]$	1-8 \supset I

4. Equivalence

a. Derive: $\sim (\sim A \& \sim B)$

1		$A \vee B$	Assumption
2		$\sim \sim A \vee B$	1 DN
3		$\sim \sim A \vee \sim \sim B$	2 DN
4		$\sim (\sim A \& \sim B)$	3 DeM

Derive: $A \vee B$

1		$\sim (\sim A \ \& \ \sim B)$		Assumption
2		$\sim \sim A \vee \sim \sim B$		1 DeM
3		$A \vee \sim \sim B$		2 DN
4		$A \vee B$		3 DN

c. Derive: $\sim (A \supset C) \supset \sim B$

1		$(A \ \& \ B) \supset C$		Assumption
2		$(B \ \& \ A) \supset C$		1 Com
3		$B \supset (A \supset C)$		2 Exp
4		$\sim (A \supset C) \supset \sim B$		3 Trans

Derive: $(A \ \& \ B) \supset C$

1		$\sim (A \supset C) \supset \sim B$		Assumption
2		$B \supset (A \supset C)$		1 Trans
3		$(B \ \& \ A) \supset C$		2 Exp
4		$(A \ \& \ B) \supset C$		3 Com

e. Derive: $A \vee (\sim B \equiv \sim C)$

1		$A \vee (B \equiv C)$		Assumption
2		$A \vee [(B \supset C) \ \& \ (C \supset B)]$		1 Equiv
3		$A \vee [(\sim C \supset \sim B) \ \& \ (C \supset B)]$		2 Trans
4		$A \vee [(\sim C \supset \sim B) \ \& \ (\sim B \supset \sim C)]$		3 Trans
5		$A \vee [(\sim B \supset \sim C) \ \& \ (\sim C \supset \sim B)]$		4 Com
6		$A \vee (\sim B \equiv \sim C)$		5 Equiv

Derive: $A \vee (B \equiv C)$

1		$A \vee (\sim B \equiv \sim C)$		Assumption
2		$A \vee [(\sim B \supset \sim C) \ \& \ (\sim C \supset \sim B)]$		1 Equiv
3		$A \vee [(C \supset B) \ \& \ (\sim C \supset \sim B)]$		2 Trans
4		$A \vee [(C \supset B) \ \& \ (B \supset C)]$		3 Trans
5		$A \vee [(B \supset C) \ \& \ (C \supset B)]$		4 Com
6		$A \vee (B \equiv C)$		5 Equiv

5. Inconsistency

a. 1		$[(E \ \& \ F) \vee \sim \sim G] \supset M$		Assumption
2		$\sim [(G \vee E) \ \& \ (F \vee G)] \supset (M \ \& \ M)$		Assumption
3		$\sim ([G \vee E] \ \& \ [F \vee G]) \supset M$		2 Idem
4		$\sim ([G \vee E] \ \& \ [G \vee F]) \supset M$		3 Com
5		$\sim ([G \vee (E \ \& \ F)] \supset M)$		4 Dist
6		$\sim ([E \ \& \ F] \vee G) \supset M$		5 Com
7		$\sim ([E \ \& \ F] \vee \sim \sim G) \supset M$		6 DN

c.	1	M & L	Assumption
	2	[L & (M & ~ S)] \supset K	Assumption
	3	~ K \vee ~ S	Assumption
	4	~ (K \equiv ~ S)	Assumption
	5	K \supset ~ S	3 Impl
	6	[(L & M) & ~ S] \supset K	2 Assoc
	7	(L & M) \supset (~ S \supset K)	6 Exp
	8	L & M	1 Com
	9	~ S \supset K	7, 8 \supset E
	10	(K \supset ~ S) & (~ S \supset K)	5, 9 &I
	11	K \equiv ~ S	10 Equiv
e.	1	~ [W & (Z \vee Y)]	Assumption
	2	(Z \supset Y) \supset Z	Assumption
	3	(Y \supset Z) \supset W	Assumption
	4	~ W \vee ~ (Z \vee Y)	1 DeM
	5	~ Z	A / ~ E
	6	~ (Z \supset Y)	2, 5 MT
	7	~ (~ Z \vee Y)	6 Impl
	8	~ ~ Z & ~ Y	7 DeM
	9	~ ~ Z	8 &E
	10	~ Z	5 R
	11	Z	5-10 ~ E
	12	Z \vee Y	11 \vee I
	13	~ ~ (Z \vee Z)	12 DN
	14	~ W	4, 13 DS
	15	~ (Y \supset Z)	3, 14 MT
	16	~ (~ Y \vee Z)	15 Impl
	17	~ ~ Y & ~ Z	16 DeM
	18	~ Z	17 &E

6. Validity

a. Derive: ~ B

	1	(R \supset C) \vee (B \supset C)	Assumption
	2	~ (E & A) \supset ~ (R \supset C)	Assumption
	3	~ E & ~ C	Assumption
	4	~ E	3 &E
	5	~ E \vee ~ A	4 \vee I
	6	~ (E & A)	5 DeM
	7	~ (R \supset C)	2, 6 \supset E
	8	B \supset C	1, 7 DS
	9	~ C	3 &E
	10	~ B	8, 9 MT

c. Derive: $\sim W \supset \sim A$

1	$A \supset [W \vee \sim (C \vee R)]$		Assumption
2	$\sim R \supset C$		Assumption
3	$\sim W$		A / \supset I
4	A		A / \sim I
5	$W \vee \sim (C \vee R)$		1, 4 \supset E
6	$\sim (C \vee R)$		3, 5 DS
7	$\sim \sim R \vee C$		2 Impl
8	$R \vee C$		7 DN
9	$C \vee R$		8 Com
10	$\sim A$		4-9 \sim I
11	$\sim W \supset \sim A$		3-10 \supset I

e. Derive: $J \supset \sim (E \vee \sim M)$

1	$\sim (J \ \& \ \sim H)$		Assumption
2	$\sim H \vee M$		Assumption
3	$E \supset \sim M$		Assumption
4	J		A / \supset I
5	$\sim J \vee \sim \sim H$		1 DeM
6	$\sim \sim J$		4 DN
7	$\sim \sim H$		5, 6 DS
8	M		2, 7 DS
9	$\sim \sim M$		8 DN
10	$\sim E$		3, 9 MT
11	$\sim E \ \& \ \sim \sim M$		10, 9 $\&$ I
12	$\sim (E \vee \sim M)$		11 DeM
13	$J \supset \sim (E \vee \sim M)$		4-12 \supset I

g. Derive: $\sim A \supset [H \supset (F \ \& \ B)]$

1	$(H \ \& \ \sim S) \supset A$		Assumption
2	$\sim B \supset \sim S$		Assumption
3	$\sim S \vee C$		Assumption
4	$C \supset F$		Assumption
5	$\sim A$		A / \supset I
6	H		A / \supset I
7	$H \supset (\sim S \supset A)$		1 Exp
8	$\sim S \supset A$		6, 7 \supset E
9	$\sim \sim S$		5, 8 MT
10	C		3, 9 DS
11	F		4, 10 \supset E
12	$\sim \sim B$		2, 9 MT
13	B		12 DN
14	$F \ \& \ B$		11, 13 $\&$ I
15	$H \supset (F \ \& \ B)$		6-14 \supset I
16	$\sim A \supset [H \supset (F \ \& \ B)]$		5-15 \supset I

7. Inconsistency

a.	1	B \vee \sim C	Assumption
	2	(L \supset \sim G) \supset C	Assumption
	3	(G \equiv \sim B) & (\sim L \supset \sim B)	Assumption
	4	\sim L	Assumption
	5	\sim L \vee \sim G	4 \vee I
	6	L \supset \sim G	5 Impl
	7	C	2, 6 \supset E
	8	\sim L \supset \sim B	3 &E
	9	\sim B	4, 8 \supset E
	10	\sim C	1, 9 DS

8.a. The rules of replacement are two-way rules. If we can derive **Q** from **P** by using only these rules, we can derive **P** from **Q** by using the rules in reverse order.

c. Suppose that before a current line **n** of a derivation, an accessible line **i** contains a sentence of the form **P** \supset **Q**. The sentence **P** \supset (**P** & **Q**) can be derived by using the following routine:

	i	P \supset Q	
	n	P	Assumption
	n + 1	Q	i , n \supset E
	n + 2	P & Q	n , n + 1 &E
	n + 3	P \supset (P & Q)	n - n + 2 \supset I