### CHAPTER THREE

Section	on 3.11	2				
<b>1.</b> a. $2^1$ c. $2^2$	= 2 = 4					
<b>2.</b> a. E	↓   ~ ~	(E & -	~ E)			
	БТ	трі				
I F	F T	FF?	ΓF			
с.		$\downarrow$				
А	J	$I \equiv [$	$J \equiv (A$	I = J		
Т	T	гт	тт	тт		
Т	F 7	TI	T T	FF		
F F	T I F I		FF I	T F		
e.					$\downarrow$	
A	н Ј	[~ A	∨ (H	$\supset$ J)]	→ (A	∨ J)
T	т т	FT	т т	ТТ	т т	ТТ
Т	T F	FT	F T	FF	Т Т	ΤF
T	FT	FT	T F	ТТ	TT	ТТ
Т	F F T T	FT	TF	TF	ТТ	ТЕ
F	TF			I I F F	I F F F	I I F F
F	FT	TF	TF	ТТ	TF	ТТ
F	FF	TF	T F	TF	F F	FF
g.			$\downarrow$			
A	B	- (A v	B) $\supset$	(~ A ∨	~ B)	
	т	тт	ТТ	FT F	FΤ	
Т	F I	ТТ	FΤ	<b>FT T</b>	ΤF	
F	Т	FFT	Т Т	ТГТ	FΤ	
F	F 7	<b>FF</b>	F T	TFT	ΤF	
i.		$\downarrow$				
В	E H	~ (E	& [H	$\supset$ (B	& E)	])
Т	ТТ	FΤ	ТТ	ТТ	ТТ	
Т	T F	FT	T F	ΤΤ	Т Т	
T	F T	TF	F T	F T	F F	
T	FF		FF	ТТ	FF	
F F	гт	TT	F T T F	К К Т Г	FT	
r F	FT		г г F Т	FF	FF	
r F	 F F		F F	TE	FF	

k.				$\downarrow$
	D	E	F	$\sim [D \& (E \lor F)] \equiv [\sim D \& (E \& F)]$
	Т	Т	Т	FTTTTT TFTFTTT
	Т	Т	F	FTTTFFFFFF
	Т	F	T	FTTFTTTFFFT
	I F	r T	r T	TFFTTTTTTTT
	F	T	F	TFF TTF F TFF TFF
	F	F	Т	TFF FTT FTFFFT
	F	F	F	TFFFFFFFFFFFFF
m				.l.
	Α	Н	I I	$  (A \lor (~A \& (H \supset I))) \supset (I \supset H)$
		T		
	Г Т	T T	T F	
	T	F	T	TTFTFFTTFFF
	T	F	F	TTFTFFTFTFTF
	F	Т	Т	FT TFT TTT TTTT
	F	T	F	FF TFF TFF TFT TFT FTT
	r F	r F	F	
	I	1	1	
<b>3.</b> a.	٨	D	C	$\downarrow \qquad \qquad$
-	A 	В	C	$\sim [\sim A \lor (\sim C \lor \sim B)]$
	F	Т	T	F TF T FT F FT
c.	Δ	в	C	$\downarrow$ (A $\supset$ B) $\vee$ (B $\supset$ C)
		D	0	
	F	Т	Т	FTTTTT
e.	A	B	C	$(A \equiv B) \lor (B \equiv C)$
-	<u> </u>	5	<u> </u>	
	F	Т	Т	FFTTTT
~				1
g.				$\checkmark$
0	Δ	R	C I	$\sim [B \supset (A \lor C)]$ & $\sim B$
0	A	В	С	$\sim [B \supset (A \lor C)] \& \sim \sim B$
	A F	B T	C T	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
	A F	B T	C T	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
i.	A F	B T R	C T	$ \begin{array}{c c} \sim [B \supset (A \lor C)] & & \sim \sim B \\ \hline \mathbf{F} \mathbf{T} \mathbf{T} \mathbf{F} \mathbf{T} \mathbf{T} \mathbf{F} \mathbf{T} \mathbf{T} \mathbf{F} \mathbf{T} \mathbf{F} \mathbf{T} \mathbf{F} \\ \downarrow \\ \sim [\alpha (A = \alpha B) = \alpha A] = (B \lor C) \end{array} $
i.	A F A	B T B	C T C	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$

<b>4.</b> a.						$\downarrow$																
	D	F	G	-	F	$\vee$	(G	$\vee$	D)													
	Т	Т	Т		Т	Т	Т	Т	Т													
	Т	Т	F		Т	Т	F	Т	Т													
	Т	F	Τ	·	F	Т	Т	Т	Т													
	Т	F	F		F	Т	F	Т	Т													
	F F	Т	1 F		Т	Т	T	T F	F F													
	F	F	г Т	.	F	Т	г Т	Г	F													
	F	F	F		F	F	F	F	F													
с.								$\downarrow$														
	D	F (	G	[F	$\vee$	(G	V D	)] &	(~	(F	$\vee$	G)	$\vee$	[~	(F	$\vee$	D)	$\vee$	~	(G	$\vee$	D)])
	Т	Т	Г	Т	Т	Т	тт	F	F	Т	Т	Т	F	F	Т	Т	Г	F	F	Т	Т	Т
	Т	ΤI	F	Т	Т	F	ΤТ	F	F	Т	Т	F	F	F	Т	Т	Т	F	F	F	Т	Т
	Т	F 7	Г	F	Т	Т	ΤТ	F	F	F	Т	Т	F	F	F	Т	Т	F	F	Т	Т	Т
	Т	FI	F	F	T	F	ТТ	T	T	F	F	F	Т	F	F	Τ̈́	Г	F	F	F	Т	T
	F	T	F	Т	Т	Т	TF	F	F	Т	Т	T F	F	F	Т	T I T I	F F	F	F T	Т	Т	F
	r F	F '	г Г	F	Т	г	гг ТF	T	r F	F	Т	r T	Т	г Т	F	т. F 1	r F	Т	I F	г Т	г Т	r F
	F	FI	F	F	F	F	FF	F	Ť	F	F	F	T	T	F	FI	F	T	Т	F	F	F
e.								$\downarrow$														
	D	F	G	-	(F	&	: G)	) ∨	[(	F	&	D	)	$\vee$	(G	8	k	D)	]			
	т	т	т	,	т	т	. т	т		т	т	т		т	т	п	г	т	-			
	T	T	F		Т	F	F	T		T	T	T		T	F	F	7	T				
	Т	F	Т	•	F	F	Т	Т		F	F	Т		Т	Т	1	Γ	Т				
	Т	F	F		F	F	F	F		F	F	Т		F	F	F	7	Т				
	F	T	T		Т	T	T	T		T	F	F		F	T	F	7	F				
	F	Т	F T		Т	' F ' F	F T	F F		T F	F F	F		F F	F T	r F	6 7	F F				
	F	F	F		F	'F	F	F		r F	г F	F		г F	F	F	7	r F				
	-	-	-		-	-	-	-		-	-	-		-	-	-		-				
g.									$\downarrow$													
C	D	F	G	[(F	&	G)	& ~	D]	V (	[(F	&	D)	&	~	G]	$\vee$	[(0	3 &	: E	) 8	sc -	~ F])
	т	т	г	т	т	т	FF	т	F	т	т	т	F	F	т	F	ſ	гт	т '	וי	 F 1	FТ
	T	T	F	Ť	F	F	FF	T	T	Ť	Ť	T	T	T	F	T	1	FF	ī		F ]	FT
	Т	F	г	F	F	Т	FF	Т	Т	F	F	Т	F	F	Т	Т	1	ГТ	Т		Г	ΓF
	Т	F ]	F	F	F	F	FF	Т	F	F	F	Т	F	Т	F	F	1	FF	Т	1	F 1	ΓF
	F	T	Γ	T	Т	Т	ТТ	F	T	T	F	F	F	F	Т	F	1	F	F	' ]	7 ] 	FT
	F F	T]	Г   Г	T F	F F	F T	F Т гт	'F' F	F F	Т	F F	F F	F	Т	Г Т	F F	1	f F F F	F	· ]	11 ] 	FT TF
	r F	F	F	r F	r F	F	г I F T	r 'F	г F	r F	r F	r F	г F	г Т	F	r F	L L	L IF F IF	r F	נ וי	с. Г'	TF
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G T T T T T T T T T T T T T T T T T T T	Image: Constraint of the second sec	$F \lor$ $F \lor$ $F \lor$ $F \top$	(G TFTFTFTFTFTFTFTTFTTTFTTTTTTTTTTTTTTTT	$ \begin{array}{c} & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & $	D) TTTFFFF $TFFTFFTFFTTFFTTFFTTFFTTFFTTFFTTFFTTFFT$	)] ~ FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF	$ \neg T F F F F F F F T (F) T T T F F F F F T T T T F F F F F T T T T T T T T F F F F T$	[F]   T   T   F     T   T   F   F   V     T   T   T   T   T     T   T   T   T   T	& T FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF	(G T F T F T F T F T F	& F F F F F F F	D) T T T F F F F F
T T F T T F F T T F F T T F F T T F F T T F F T T F F T T F F T T F F T T F F T T F F T T F F T T F F F F T T F F F F T T F F F T T F F F T T F F F T T F F F F T T F F F F T T F F F F T T F F F F F T T F F F F T T F F F F T T F F F F T T F F F F F T T F		r T r T r T r T r T r T r T r T r T r T	$\begin{array}{c} T \\ F \\ T \\ F \\ T \\ F \\ T \\ F \\ T \\ T \\$	T T T T F F T T F F T T F F T T F F F T T F F F T T F F F T T F F F T T F F F T T F F F F T T F	TTTFFFF FFF ⊃ FFTTFFTTFFT	~ FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF	T F F F F F F T T T F F F F T T T F F F F T T T T T F F F T	T T F F T T F F V T T T T T T T T T T T	T F F F F F F F F F F F T T T T T T T T	T F T F T F T F	T F F F F F F F	T T T F F F F F
		T T T T T T T T T T T T T T T T T T T	$ \begin{array}{c} \mathbf{I} \\ \mathbf{F} \\ \mathbf{T} \\ \mathbf{F} \\ \mathbf{F} \\ \mathbf{T} \\ \mathbf{F} \\ \mathbf{T} \\ \mathbf{F} \\ \mathbf{T} \\ \mathbf{F} \\ \mathbf{T} \\ \mathbf{T} \\ \mathbf{T} \\ \mathbf{T} \\ \mathbf{F} \\ \mathbf{T} \\ \mathbf$	$\begin{bmatrix} G \\ T \\ T \\ F \\ F \\ F \\ T \\ F \\ F \\ T \\ F \\ F$	I T T T F F F F F F F F T T F F F T T F F F T T F F F T T F F F F T T F F F F F F F F F F F F F F F F F F F T T T F F F F T T T F F F F T T T T T F F F T T T T F F F F T T T T T F F F T	~ FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF	F F F F F F F F T T T T F F F F F T T T T T F F F F T T T	T F F T T F F V T T T T T T T T T T T T	F F F F F F F F F F F F F F F F F F F	I F T F T F T F F	I F T F F F F F F	T T T F F F F F
F         T         F           F         T         F           T         F         T           T         F         T           T         F         T           T         F         T           T         F         T           T         F         T           T         F         T           T         F         F           T         F         F           T         F         F		T T T T T T T T T T T T T T T T T T T	$ \begin{array}{c} \mathbf{F} \mathbf{T} \mathbf{T} \mathbf{F} \mathbf{T} \mathbf{T} \mathbf{F} \mathbf{T} \mathbf{T} \mathbf{T} \mathbf{F} \mathbf{T} \mathbf{T} \mathbf{T} \mathbf{F} \mathbf{T} \mathbf{T} \mathbf{T} \mathbf{T} \mathbf{T} \mathbf{T} \mathbf{T} T$	$\begin{bmatrix} G \\ T \\ T \\ F \\ F \\ F \\ T \\ F \\ F \\ T \\ F \\ F$	TTFFFF FFF FFTTFFTTFFT TFFTT	~ FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF	F F F F F F T T T F F F F F T T T T F F F F T T T	T F F T T F F F T T T T T T T T T T T T	F F F F F F F F F F T T T T T T T T T T	F T F T F T F T F	F T F F F F F F F	T T F F F F F F
		F T T T T T T T T T T T T T T T T T T T	$\begin{array}{c} \mathbf{T} \\ \mathbf{F} \\ \mathbf{T} \\ \mathbf{F} \\ \mathbf{F} \\ \mathbf{F} \\ \mathbf{F} \\ \mathbf{T} \\ \mathbf{F} \\ \mathbf{T} \\ \mathbf{T} \\ \mathbf{F} \\ \mathbf{T} \\ \mathbf{T} \\ \mathbf{F} \\ \mathbf{T} \\ \mathbf{T} \\ \mathbf{T} \\ \mathbf{F} \\ \mathbf{T} \\ $	$ \begin{array}{c} \mathbf{T} \mathbf{T} \mathbf{T} \mathbf{F} \mathbf{F} \mathbf{T} \mathbf{F} \mathbf{F} \mathbf{F} \mathbf{T} \mathbf{F} \mathbf{F} \mathbf{F} \mathbf{T} \mathbf{T} \mathbf{F} \mathbf{F} \mathbf{F} \mathbf{F} \mathbf{T} \mathbf{T} \mathbf{F} \mathbf{F} \mathbf{F} \mathbf{T} \mathbf{T} \mathbf{F} \mathbf{F} \mathbf{F} \mathbf{T} \mathbf{T} \mathbf{T} \mathbf{T} \mathbf{F} \mathbf{F} \mathbf{T} \mathbf{T} \mathbf{T} \mathbf{T} \mathbf{T} \mathbf{T} \mathbf{T} T$	TTFFFF FFF ⊃ FFTTFFTTFFT	~ FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF	F F F F T T T T F F F F T T T T T T F F F T	F F T T F F F V T T T T T T T T T T T T	F F F F F F F T T T T T T T T T T T T T	T F T F T F F	T F F F F F F	T F F F F F
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T F F F T T F F T T F F T T F F F T T F F F T T F F F T T F F F T T F F F T T F		C T T T F F F F F F T F T F T F T F T F	$ \begin{array}{c} \mathbf{T} \\ \mathbf{F} \\ \mathbf{T} \\ \mathbf{F} \\ \mathbf{F} \\ \mathbf{F} \\ \mathbf{T} \\ \mathbf{T} \\ \mathbf{F} \\ \mathbf{T} \\ \mathbf{T} \\ \mathbf{F} \\ \mathbf{T} \\ \mathbf{T} \\ \mathbf{T} \\ \mathbf{F} \\ \mathbf{T} \\ \mathbf$	T F T F F T T F F T T F F F T T F F F T T F F F T T F F F T T F F F T T F	F F F F F F T T F F T T F F T T F F T T F F T T F F T T F F T T T F T	~ FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF	<b>F</b> <b>F</b> <b>T</b> (F <b>T</b> <b>T</b> <b>T</b> <b>T</b> <b>T</b> <b>T</b> <b>F</b> <b>F</b> <b>F</b> <b>F</b> <b>T</b>	T F F T T T T T T T T T T	<b>F</b> <b>F</b> <b>F</b> <b>F</b> <b>D</b> <b>T</b> <b>T</b> <b>T</b> <b>T</b> <b>T</b> <b>T</b> <b>T</b> <b>T</b> <b>T</b> <b>T</b>	T F T F	F F F F	F F F F
F T F F T T F F T T F F T T F F F T T F F F T T F F F T T F F F T T F		T T T F F F F F F F F F F F F F F F F F	F $T$ $F$ $T$	F T F F T T F F T T F F F T T F F F T T F F F T T F	F F F F F T F F T F F T T F F T T F F T T F F T T F F T T T F T	~ FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF	<b>F</b> <b>F</b> <b>T</b> (F <b>T</b> <b>T</b> <b>T</b> <b>T</b> <b>T</b> <b>T</b> <b>F</b> <b>F</b> <b>F</b> <b>F</b> <b>F</b> <b>T</b>	T F F V T T T T T T T T T T T T T T T T	F F D) T T T T T T T F	F T F	F F F	F F F
T T F G G T T T F F T T F F T T F F F T T F F F T T F F F T T F F F T T F	S S F T F T F T F T F T F T F T F T F T	F T F F F F F T F T F T F T F T F T F T	$\begin{array}{c} \mathbf{T} \\ \mathbf{F} \\ \rightarrow \\ \mathbf{F} \\ \mathbf{T} $	T F F T T F F T T F F T T F F T T F F	$ \begin{array}{c} \mathbf{F} \\ \mathbf{T} \\ \mathbf{F} \\ \mathbf{F} \\ \mathbf{F} \\ \mathbf{T} \\ \mathbf{F} \\ \mathbf{F} \\ \mathbf{F} \\ \mathbf{T} \\ \mathbf{F} \\ \mathbf$	~ FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF	F T (F T T T T F F F F T T	F F T T T T T T T T T T	F F D) T T T T T T F	T F	FF	FF
F G T T F F T F F T F F T F F F F F F	S T F T F T F T F T F T F T F T F T	F F S T F T F T F T F T F T F T F T F	$F \rightarrow $ $T \rightarrow $	F [G T T F F T T F F T T F F T T F F	F F F T T F F T T F F T T F F T T F F T	~ FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF	T (F T T T T F F F F T T	F	F D) T T T T T T T F	] 	F	F
G T T F F T F F T T F F T T F F F T T F	S T F T F T F T F T F T F T	S T F T F T F T F T F T F T F T F T	$ \downarrow $ $ \neg $ $ F T T T F T T T F T T T T T T T T T T $	[G T F F T T F F T T F F F	⊃ FFT FFT FFT FFT	~ FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF	(F T T T T F F F T T	V T T T T T T T T T T	D) T T T T T T T T F			
G T T F F T F T F T T F F T T F F T T F F T	S T F T F T F T F T F T F T	S T F T F T F T F T F T F T F T F T F	$\begin{array}{c} \downarrow \\ \neg \end{array}$ <b>F T T T F T T T F T T T T T T T T T T</b>	[G T F F F T T F F T T F F F	⊃ F F T T F F T F F T T F F T	~ FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF	(F T T T T F F F F T T	V T T T T T T T T T T	D) T T T T T T T F	0]		
G T T F F F T F F T F F F T F F F F F F	S T F T F T F T F T F T F T F T	S T F T F T F T F T F T F T F T F T	→ F T <p< td=""><td>[G T T F F T T F F T T F F F T F F</td><td>⊃ F F T F F T F F T F F T</td><td>~ FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF</td><td>(F T T T T F F F T T</td><td>Y     T</td><td>D) T T T T T T T F</td><td></td><td></td><td></td></p<>	[G T T F F T T F F T T F F F T F F	⊃ F F T F F T F F T F F T	~ FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF	(F T T T T F F F T T	Y     T	D) T T T T T T T F			
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	F F F F F F F F F F T F T	F T F T F T F T F T F T F T F T	r T T T F T T T T T T T	T F F T T F F T T F F	F T T F F T T F F T T F F T	r F F F F F F F F F F F F F F F F F F F	T T T F F F F T T	T T T T T T T T T	T T T T T T T F			
	r T F T F T F T F T F T	T F T F T F T F T F T F T	T T F T T T T T T T	I F F T T F F T T F F	r T F F T F F T F F T F T	r F F F F F F F F F F F F F F F F F F F	T T F F F F T T	T T T T T T T T	T T T T T T F			
F F T F F F F F F F	F F T F T F T F T F	F F T F T F T F T F T	T F T T F T T T T	F F T F F F T F F F	T F F T T F F T T F T	r F F F F F F F F F F F F F	T F F F F T T	T T T T T T T T	T T T T T F			
F T F F T T T F T T F	F T F T F T F T F	F T F T F T F T F T	T F T T F T T T	F T T F F T F F F	T F F T T F F T	F F F F F F F F F F	T F F F F T T	T T T T T T	T T T T F			
T F F T T T F F	T F T F T F T F	T   F   T   F   T   F   T	F T T F T T	T F F T F F F	F F T F F T	F F F F F F F F F	F F F T T	T T T T T	T T T F			
T F F T T T F F F	F T F T F T F T	F   T   F   T   F   T	T T F T T	T F F T F F	F T F F T	F F F F F F F	F F F T T	T T T T	T T T F			
F F T T T F F F	T F T F T F T	T   F   T   F   T   F	T F T T	F F T T F F	T T F T T	F F F F	F F T T	T T T	T T F			
F T T T F F F	F T F T F T	F T F T F T	T F T T	F T T F F	T F F T	F F F	F T T	T T	T F			
T T F F	T F T F T	T F T F T	F T T T	T T F F	F F T	F F	T T	Т	F			
T F F	F T F T	F T F T	T T T	T F F	F T	F	т					
F F	T F T	T F T	T T	F F	Т	Б	-	1	F			
F	F T	F T	Т	F		г	Т	Т	F			
_	T	T	_		- 1	F	Т	Т	F			
т			Т	т	т	т	F	F	F			
Ť.	F	F	Ť	Ť	Ť	Ť	F	F	F			
- F	T	Т	Ť	F	T	т	F	F	F			
r F	F	F	т	r F	т	т	r F	F	F			
г	r	ſ	I	г	1	I	г	г	г			
		$\downarrow$										
S	D	=	(P	&	S)							
Т	Т	Т	Т	Т	Т							
F	Т	F	Т	F	F							
Т	Т	F	F	F	Т							
F	Т	F	F	F	F							
т '	F	F	Ť	Ť	T							
- -	F	Ť	Ť	Ē	F							
Π.		т	F	TC IC	T							
T	r r	I T	r	r	1 F							
T		1	ľ	r	r							
	F T F T F T F	F     T       T     T       F     T       F     F       F     F       F     F       F     F       F     F	F       T       F         T       T       F         F       T       F         T       F       F         T       F       T         T       F       T         T       F       T         F       F       T         F       F       T	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	F       T       F       T       F       F         T       T       F       F       F       T         F       T       F       F       F       F         T       F       F       T       T       T         F       F       T       T       T       F         T       F       F       T       T       F         T       F       T       T       F       F         T       F       T       F       F       T         F       F       T       F       F       T         F       F       T       F       F       T         F       F       T       F       F       F         F       F       T       F       F       F         F       F       T       F       F       F         F       T       F       F       F       F         F       T       F       F       F       F         F       T       F       F       F       F         F       T       F       F       F	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	F       T       F       T       F       F         T       T       F       F       T       T         F       T       F       F       F       T         T       F       F       T       T       T         F       F       T       T       T       T         F       F       T       T       F       F         T       F       T       T       F       F         T       F       T       F       F       T         F       F       T       F       F       T         F       F       T       F       F       T         F       F       T       F       F       F         F       F       T       F       F       F	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$

g.						$\downarrow$							
D	F	G	Р	R	Р	$\supset$	(F	$\supset$	[~ (D	$\vee$	G)	&	R])
T	Т	Т	Т	Т	Т	F	Т	F	FΤ	Т	Т	F	Т
Т	Т	Т	Т	F	T	F	Т	F	FΤ	Т	Т	F	F
Т	Т	Т	F	Т	F	Т	Т	F	FΤ	Т	Т	F	Т
Т	Т	Т	F	F	F	Т	Т	F	FΤ	Т	Т	F	F
Т	Т	F	Т	Т	T	F	Т	F	FΤ	Т	F	F	Т
Т	Т	F	Т	F	T	F	Т	F	FΤ	Т	F	F	F
Т	Т	F	F	Т	F	Т	Т	F	FΤ	Т	F	F	Т
Т	Т	F	F	F	F	Т	Т	F	FΤ	Т	F	F	F
Т	F	Т	Т	Т	T	Т	F	Т	FΤ	Т	Т	F	Т
Т	F	Т	Т	F	T	Т	F	Т	FΤ	Т	Т	F	F
Т	F	Т	F	Т	F	Т	F	Т	FΤ	Т	Т	F	Т
Т	F	Т	F	F	F	Т	F	Т	FΤ	Т	Т	F	F
Т	F	F	Т	Т	T	Т	F	Т	FΤ	Т	F	F	Т
Т	F	F	Т	F	T	Т	F	Т	FΤ	Т	F	F	F
Т	F	F	F	Т	F	Т	F	Т	FΤ	Т	F	F	Т
Т	F	F	F	F	F	Т	F	Т	FΤ	Т	F	F	F
F	Т	Т	Т	Т	Т	F	Т	F	FF	Т	Т	F	Т
F	Т	Т	Т	F	Т	F	Т	F	FF	Т	Т	F	F
F	Т	Т	F	Т	F	Т	Т	F	FF	Т	Т	F	Т
F	Т	Т	F	F	F	Т	Т	F	FF	Т	Т	F	F
F	Т	F	Т	Т	Т	Т	Т	Т	ΤF	F	F	Т	Т
F	Т	F	Т	F	T	F	Т	F	ΤF	F	F	F	F
F	Т	F	F	Т	F	Т	Т	Т	ΤF	F	F	Т	Т
F	Т	F	F	F	F	Т	Т	F	ΤF	F	F	F	F
F	F	Т	Т	Т	Т	Т	F	Т	FF	Т	Т	F	Т
F	F	Т	Т	F	Т	Т	F	Т	FF	Т	Т	F	F
F	F	Т	F	Т	F	Т	F	Т	FF	Т	Т	F	Т
F	F	Т	F	F	F	Т	F	Т	FF	Т	Т	F	F
F	F	F	Т	Т	T	Т	F	Т	ΤF	F	F	Т	Т
F	F	F	Т	F	T	Т	F	Т	ΤF	F	F	F	F
F	F	F	F	Т	F	Т	F	Т	ΤF	F	F	Т	Т
F	F	F	F	F	F	т	F	т	ТБ	F	F	F	F

# Section 3.2E

1.a. Truth-functionally indeterminate

$$\begin{array}{c|c} & \downarrow \\ A & \sim A \supset A \\ \hline T & F T & T & T \\ F & T F & F & F \end{array}$$

c. Truth-functionally true

А	(A	=	~ A)	$\downarrow$	~ (A	=	~ A)
T	T	F	F T	T	T T	F	F T
F	F	F	T F	T	T F	F	T F

e. Truth-functionally indeterminate

B	D	(- B	80		$\downarrow$	т. (В	V	D)
D	D	(~ D	æ	~ D)	~	~ (D	v	D)
Т	Т	FΤ	F	FΤ	F	FΤ	Т	Т
Т	F	FT	F	ΤF	F	FΤ	Т	F
F	Т	TF	F	FΤ	F	FF	Т	Т
F	F	TF	Т	ΤF	Т	ΤF	F	F

g. Truth-functionally indeterminate

А	В	С	[(A	$\vee$	B)	&	(A	$\vee$	C)]	↓ ⊃	~ (B	&	C)
Т	Т	Т	Т	Т	Т	Т	Т	Т	Т	F	FΤ	Т	Т
Т	Т	F	T	Т	Т	Т	Т	Т	F	Т	ТТ	F	F
Т	F	Т	Т	Т	F	Т	Т	Т	Т	Т	ΤF	F	Т
Т	F	F	Т	Т	F	Т	Т	Т	F	Т	ΤF	F	F
F	Т	Т	F	Т	Т	Т	F	Т	Т	F	FΤ	Т	Т
F	Т	F	F	Т	Т	F	F	F	F	Т	ТТ	F	F
F	F	Т	F	F	F	F	F	Т	Т	Т	ΤF	F	Т
F	F	F	F	F	F	F	F	F	F	Т	ΤF	F	F

i. Truth-functionally true

					$\downarrow$				
J	Κ	(J	$\vee$	~ K)	=	~ ~	(K	$\supset$	J)
Т	Т	Т	Т	FΤ	Т	ΤF	Т	Т	Т
Т	F	Т	Т	ΤF	Т	ΤF	F	Т	Т
F	Т	F	F	FΤ	Т	FΤ	Т	F	F
F	F	F	Т	ΤF	Т	ΤF	F	Т	F

k. Truth-functionally true

А	D	[(A	$\vee$	~ D)	&	~ (A	&	D)]	$\downarrow$	~ D
Т	Т	Т	Т	FΤ	F	FТ	Т	Т	Т	FΤ
Т	F	Т	Т	ΤF	Т	ТТ	F	F	Т	ΤF
F	Т	F	F	FΤ	F	ΤF	F	Т	Т	FΤ
F	F	F	Т	ΤF	Т	ТБ	F	F	Т	ΤF

**2.**a. Not truth-functionally true

F	F	F	F	F	F	ΤF	F	F
F	Η	(F	$\vee$	H)	$\vee$	(~ F	=	H)
					$\downarrow$			

# c. Truth-functionally true

				$\downarrow$					
А	В	С	~ A	$\supset$	[(B	&	A)	$\supset$	C]
Т	Т	Т	FΤ	Т	Т	Т	Т	Т	Т
Т	Т	F	FΤ	Т	Т	Т	Т	F	F
Т	F	Т	FΤ	Т	F	F	Т	Т	Т
Т	F	F	FΤ	Т	F	F	Т	Т	F
F	Т	Т	ΤF	Т	Т	F	F	Т	Т
F	Т	F	ΤF	Т	Т	F	F	Т	F
F	F	Т	ΤF	Т	F	F	F	Т	Т
F	F	F	TF	Т	F	F	F	Т	F

e. Truth-functionally true

# 3.a. Truth-functionally false

				$\downarrow$			
D	(B	≡	D)	&	(B	≡	~ D)
Т	Т	Т	Т	F	Т	F	FТ
F	Т	F	F	F	Т	Т	ΤF
Т	F	F	Т	F	F	Т	FΤ
Г	Г	т	Г	F	F	F	тг
	D T F T	D (B T T F T T F F	$\begin{array}{c c} D & (B \equiv \\ \hline T & T & T \\ F & T & F \\ T & F & F \\ F & F & T \end{array}$	$\begin{array}{c ccc} D & (B \equiv D) \\ \hline T & T & T & T \\ F & T & F & F \\ T & F & F & T \\ F & F & T & F \end{array}$	$ \begin{array}{c c} & \downarrow \\ D & (B \equiv D) & \& \\ \hline T & T & T & T & F \\ F & T & F & F & F \\ T & F & F & T & F \\ F & F & T & F & F \\ F & F & T & F & F \\ \end{array} $	$\begin{array}{c cccc} & \downarrow \\ D & (B \equiv D) & \& & (B \\ \hline T & T & T & T & F & T \\ F & T & F & F & F & T \\ T & F & F & T & F & F \\ F & F & T & F & F & F \\ F & F & T & F & F & F \\ \end{array}$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$

c. Not truth-functionally false

Т	Т	Т	Т	Т	Т	Т
Α	В	Α	=	(B	≡	A)
			$\downarrow$			

e. Not truth-functionally false

F	Т	F	Т	Т	F	F	Т	ΤF
С	D	[(C	$\vee$	D)	=	C]	$\supset$	~ C
							$\downarrow$	

**4.**a. False. For example, while '(A  $\supset$  A)' is truth-functionally true, '(A  $\supset$  A) & A' is not.

c. True. There cannot be any truth-value assignment on which the antecedent is true and the consequent false because there is no truth-value assignment on which the consequent is false.

e. False. For example, although '(A & ~ A)' is truth-functionally false, 'C  $\vee$  (A & ~ A)' is not.

g. True. Since a sentence  $\sim \mathbf{P}$  is false on a truth-value assignment if and only if  $\mathbf{P}$  is true on the truth-value assignment,  $\mathbf{P}$  is truth-functionally true if and only if  $\sim \mathbf{P}$  is truth-functionally false.

i. False. For example, '(A  $\lor \sim A$ )' is truth-functionally true, but '(A  $\lor \sim A$ )  $\supset B$ ' is truth-functionally indeterminate.

5.a. On every truth-value assignment, **P** is true and **Q** is false. Hence  $\mathbf{P} \equiv \mathbf{Q}$  is false on every truth-value assignment. Therefore  $\mathbf{P} \equiv \mathbf{Q}$  is truth-functionally false.

c. No. Both 'A' and '~ A' are truth-functionally indeterminate, but 'A  $\vee$  ~ A' is truth-functionally true.

### Section 3.3E

1.a. Not truth-functionally equivalent

		$\downarrow$				$\downarrow$											
А	B	~ (	А	& 1	B) ·	~ (A	$\vee$	B)	)								
Т	Т	F	Т	T 7	Γ	FΤ	Т	Т									
Т	F	Т	Т	F I	F ]	FΤ	Т	F									
F	Т	Т	F	F 7	Γ	FF	Т	Т									
F	F	Т	F	FJ	F'	Γ F	F	F									
c. Tru	ıth-f	unct	iona	ally (	equi	valei	nt	.1.									
TT	17	172	*	тт			17	*	тт								
н	К	K	=	Н		~	ĸ	=	~ H								
Т	Т	Т	Т	Т		F	Т	Т	FΤ								
Т	F	F	F	Т		Т	F	F	FΤ								
F	т	т	F	F		F	т	F	ТБ								
F	F	F	T	F		T	F	т	ΤF								
e. Tru	uth-f	unct	iona	ally o	equi	valei	nt	-					I				
Б		10			$\downarrow$					10			$\downarrow$				
F	G	(G	$\supset$	F)	$\supset$	(F	$\supset$	G)		(G	=	F)	$\vee$	(~ F	$\vee$	G)	
т	т	т	т	т	т	т	т	т		т	т	т	т	БТ	т	т	
Т	F	I F	T	Т	F	Т	I E	F		I F	F	T	F	гі	F	F	
I T	r T	r	I T	I T	r	I F	r	r		r	r	1	r	r I TT	r	r	
F	1	T	F	F	T	ľ	T	T		T	F	F	1	TF	T	1	
F	F	F	T	F		F	T	_F_		F	T	_F_	_T_	TF	T	F	

g. N	ot ti	rutł	n-fu	nct	ior	nall	y e	quiv	ale	nt										
								$\downarrow$											$\downarrow$	
Н	J	K		~ (	Η	&	J)	=	(	J	=	~ K	)		(	Ή	&	J)	$\supset$	~ K
T	Т	Т		F	Т	Т	Т	Т	]		F	FΤ				Т	Т	Т	F	FΤ
T	Т	F		F '	Т	Т	Т	F	]		Т	ΤF				Т	Т	Т	Т	ΤF
Т	F	Т		Т	Т	F	F	Т	]	7	Т	FΤ				Т	F	F	Т	FΤ
Т	F	F		Т	Т	F	F	F	]	7	F	ΤF				Т	F	F	Т	ΤF
F	Т	Т		Т	F	F	Т	F	1		F	FΤ				F	F	Т	T	FT
F	T	F		T	F	F	T	T	1		Т	TF				F	F	Т	T	TF
F	F	Т		Т	F	F	F	Т	1	7	T F	FT				F	F	F	Т	FT
r	г	r	I	1	г	г	г	г	1	2	г	Ir				г	г	г	1	1 Г
i. N	ot t	rutł	n-fu	nct	ior	nall	y e	quiv	ale	nt										
									$\downarrow$										$\downarrow$	
А	С	D	[A	$\vee$	~	(D	&	C)]	$\supset$	~	D		[D	$\vee$	~	(A	&	C)]	$\supset$	~ A
T	Т	Т	Т	Т	F	Т	Т	Т	F	F	Т		Т	Т	F	Т	Т	Т	F	FΤ
Т	Т	F	Т	Т	Т	F	F	Т	Т	Т	F		F	F	F	Т	Т	Т	Т	FΤ
Т	F	Т	Т	Т	Т	Т	F	F	F	F	Т		Т	Т	Т	Т	F	F	F	FΤ
Т	F	F	Т	Т	Т	F	F	F	Т	Т	F		F	Т	Т	Т	F	F	F	FΤ
F	Т	T	F	F	F	Т	Т	Т	Т	F	Т		Т	Т	Т	F	F	Т	Т	ΤF
F	Т	F	F	Т	Т	F	F	Т	Т	Т	F		F	Т	Т	F	F	Т	Т	ΤF
F	F	Т	F	Т	Т	Т	F	F	F	F	T		Т	Т	Т	F	F	F	Т	TF
F	F	F I	F	Т	Т	F	F	F	Т	Т	F		F	Т	Т	F	F	F	Т	ТЕ
k. N	ot t	rutł	n-fu	nct	ior	nall	v e	quiv	ale	nt										
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F	G	н		F	↓ ∨	~	(G	V	~	H)			(F	Ŧ	=	~	F)	↓ V	G	
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Т	Т	Т		Т	Т	F	Т	Т	F	Т			]	Г	F	F	Т	Т	Т	
Т	Т	F		Т	Т	F	Т	Т	Τ	F			]	F	Т	F	Т	Т	T	
	<u>F</u>	<u>T</u>	_	T	<u>T</u>	<u>T</u>	F	<u>F</u>	F	T T			1	<u> </u>	F	F	T	F	F	
Т	F	F		T E	Т	F	F	T	T	F			]	ť.	T	F	T E	Т	F	
F	I T	I F		F F	F	F F	T	I T	r T	I F			1	L F	I F	I T	F F	I T	I T	
F	F	г		г F	г	г	F	F	F	г Т			נ ר	r r	г	Т	г F	т	г Г	
F	F	F		F	F	F	F	Т	Т	F			l	F	F	т	F	F	F	
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<b>2.</b> a. T	ruth	-fur	nctio	ona	ally	eq	uiv	aler	ıt											
				$\downarrow$						$\downarrow$										
G	Η		G	$\vee$	Н			~	G	$\supset$	Н	I -								
Т	Т		Т	Т	Т			F	Т	Т	Т									
Т	F		Т	Т	F			F	Т	Т	F									
F	Т		F	Т	Т			Т	F	Т	Т	,								
F	F		F	F	F			T	F	F	F									

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c. Tr	uth-	func	tiona	ally (	equi	vale	ent							
Δ	р	(Г	) =	<b>A</b> )	v R	р			р	* &	Δ			
	D	(1		11)	a				D					
Т	Т	Г	Т	Т	Т	Т			Т	Т	Т			
Т	F	I	F	Т	F	F			F	F	Т			
F	Т	Г	F	F	F	Т			Т	F	F			
F	F	I	T	F	F	F			F	F	F			
e. No	ot tru	uth-f	unct	tiona	ally e	equ	ival	ent						
		I.			,	1			I					
٨		¥ 	/		_ ,	• `		Ì	↓ (	_		<b>A</b> )		
A	A	. =	(~	A	= }	4)		-	~ (A		~	A)		
Т	Т	' F	F	т	F 1	Г		r	гт	F	F	т		
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<b>3.</b> a. No	ot tri	uth-f	unct	iona	allv e	eau	ival	ent						
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М	: TI	he n	loon	wil	l shi	ne	brig	rhtl	v.					
							····6		/-					
-		1	-	$\downarrow$		_					$\downarrow$		_	-
С	М	N	С	$\vee$	(N	&	M)	)		М	=	(N	&	~ C)
	т	т	т	т	т	т	т			т	Б	т	Б	БТ
	<u>і</u> т	I F	<u>і</u> т	<u>і</u> т	 	1 F	<u>і</u> т			<u>і</u> т	<u>г</u> г	 F	r F	F I F T
Т	I F	r T	Т	Т	г	r F	F			I F	г	г	г Г	Г I ГT
I	r		T	T	I T	r	г			r	T	I T	r	F I F T
	r	r T	I	I	r	ľ	F			ľ	I	r	r	r I TT
r T	I		F	I	I	I	I			I	I	I	I	
F	T	F	F	F	F	F	T			T	F	F	F	TF
F	F	T	F	F	T	F	F			F	F	T	T	TF
F	F	F	F	F	F	F	F			F	Т	F	F	ΤF
т	.1 .	c		11		1								
c. Ir	ruth-1	func	tiona	ally	equi	vale	ent							
D:	Th	ne D	aily I	Hera	ld re	epo	rts (	on	our	anti	ics.			
A:	O	ır ar	ntics	are	effe	ectiv	e.							
	_	. –	$\downarrow$					$\downarrow$	_					
А	D	D	$\supset$	А		~	·Α	$\supset$	~ D	)				
	T	T	-	T			י חדי	- m	TE CE	-				
T	ľ		T	T.		ł		T	FI	,				
Т	F	F	Т	Т		ł		Т	TF					
F	T		F	F		1	F	F	FT					
F	F	<b>F</b>	Т	F		]	F	Т	ΤF					

e. Not truth-functionally equivalent

- M: Mary met Tom.
- L: Mary liked Tom.
- G: Mary asked George to the movies.

						$\downarrow$					$\downarrow$	
G	L	М	(M	&	L)	$\supset$	~ G	(M	&	~ L)	$\supset$	G
Т	Т	Т	Т	Т	Т	F	FΤ	Т	F	FΤ	Т	Т
Т	Т	F	F	F	Т	Т	FΤ	F	F	FΤ	Т	Т
Т	F	Т	Т	F	F	Т	FΤ	Т	Т	ΤF	Т	Т
Т	F	F	F	F	F	Т	FΤ	F	F	ΤF	Т	Т
F	Т	Т	Т	Т	Т	Т	ΤF	Т	F	FΤ	Т	F
F	Т	F	F	F	Т	Т	ΤF	F	F	FΤ	Т	F
F	F	Т	Т	F	F	Т	ΤF	Т	Т	ΤF	F	F
F	F	F	F	F	F	Т	ΤF	F	F	ΤF	Т	F

**4.**a. Yes. **P** and **Q** have the same truth-value on every truth-value assignment. On every truth-value assignment on which they are both true,  $\sim$  **P** and  $\sim$  **Q** are both false, and on every truth-value assignment on which they are both false,  $\sim$  **P** and  $\sim$  **Q** are both true. It follows that  $\sim$  **P** and  $\sim$  **Q** are truth-functionally equivalent.

c. If **P** and **Q** are truth-functionally equivalent then they have the same truth-value on every truth-value assignment. On those assignments on which they are both true, the second disjunct of  $\sim \mathbf{P} \vee \mathbf{Q}$  is true and so is the disjunction. On those assignments on which they are both false, the first disjunct of  $\sim \mathbf{P} \vee \mathbf{Q}$  is true and so is the disjunct of  $\sim \mathbf{P} \vee \mathbf{Q}$  is true on every truth-value assignment.

#### Section 3.4E

1.a. Truth-functionally consistent

				$\downarrow$			$\downarrow$			$\downarrow$	
А	В	С	A	$\supset$	В	В	$\supset$	С	А	$\supset$	С
_											_
Т	Т	Т	Т	Т	Т	Т	Т	Т	Т	Т	Т
Т	Т	F	Т	Т	Т	Т	F	F	Т	F	F
Т	F	Т	T	F	F	F	Т	Т	Т	Т	Т
Т	F	F	T	F	F	F	Т	F	Т	F	F
F	Т	Т	F	Т	Т	Т	Т	Т	F	Т	Т
F	Т	F	F	Т	Т	Т	F	F	F	Т	F
F	F	Т	F	Т	F	F	Т	Т	F	Т	Т
F	F	F	F	Т	F	F	Т	F	F	Т	F

c. Truth-functionally inconsisten	t	
, 1	I	I
H J L $  \sim [J \lor (H \supset L)]$	$L \equiv (\sim J \lor \sim H)$	$\stackrel{\checkmark}{H} \equiv (J \lor L)$
ттт втт ттт	т	тттт
	F T F T F F T	тт тт <b>ғ</b>
	T T TF TFT	T T F T T
	FFTFTFT	TFFFF
FTT FTT FTT	ΤΤ ΕΤΤΤΕ	FFTTT
FTF FTT FTF	FFFTTTF	FFTTF
FFT FFT FTT	ΤΤ ΤΓΤΤΓ	FFFTT
FFF FFT FTF	F F T F T T F	FT FFF
e. Truth-functionally inconsisten	t	
, 	1	
	↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓	
	~J ~ П	
тт тттт	FT FT	
TF FTFTT	TF FT	
FT TTFF	FT TF	
FFFFFF	TF TF	
g. Truth-functionally consistent		
k k k		
A B C   A B C		
T T T T T T		
T T F   T T F		
T F T   T F T		
FTTFTT		
FTFFTF		
FFT FFT		
F F F   F F F		
1. Iruth-functionally consistent		
$\downarrow$	$\downarrow$	$\downarrow$
A B C $ $ (A & B) $\vee$ (C	$\supset$ B) ~ A	~ B
	TT FT	FT
TET TIT		Г I Т Е
TEE TEET	rr fl TE ET	1 F T F
FTT FTT	і Г Г Г Г Г Т Т Т Т Т Т Т Т	гг ГТ
	лі ії ТТ ТБ	гі ГТ
		TF
FFF FFFTF		
		<b>* *</b>

SOLUTIONS TO SELECTED EXERCISES ON P. 100 31

<b>2.</b> a.	Tr	uth	fun	ction	ally	cor	sist	ent						
<b>_</b>			run	ction	1	con	10100	circ						
	р	р	Б	L D	↓ _	(D	_	E)				D	↓ 0.	D
	В	D	E	В		(D		E)			~ .	D	80	Б
	Т	F	Т	T	Т	F	Т	Т			Т	F	Т	Т
с.	Tr	uth	fun	ction	ally	con	sist	ent						
					$\downarrow$							$\downarrow$		
	F	J	Κ	F	$\supset$	(J	$\vee$	K)			F	=	~	J
		-									-	-		
	T	F	Т	· .I.	Т	F	Т	Т			T	I.	Т	F
P	$\mathbf{Tr}$	uth	fun	ction	ally	con	rict	ent						
с.	11	uun	-iun	cuon	any		15150	cint						
	٨	р			р	\ ↓	- /	D		D)			1	
	A	D	(.		D	) =	= (·	~ D	~	D)			F	-
	Т	Т		т т	Т	Т	']	FΤ	Т	Т			]	Г
<b>3.</b> a.	Tr	uth	fun	ction	ally	inc	onsi	sten	nt					
	S:	SI	bace	is in	ifini	itely	divi	sibl	e.					
	Z:	Ze	eno'	s par	ado	oxes	are	con	npe	lling	g.			
	C:	Ze	eno'	s par	ado	oxes	are	con	nvin	cing	ŗ.			
				1	. .						,			
	С	S	Ζ	S	Ď	Ζ		~	- (C	$\vee$	Z	)		Š
									`					
	Т	Т	Т	T	Т	Т		I	F T	T	Т			Т
	Т	Т	F		F	F		ł	f T z T	Т	F			Т
	I T	r F	I F	r F	т Т	I F		1 T	! I 7 Т	І Т	I F			r F
	F	г	г	T	Т	г		1	F F	T	г Т			Г
	F	Ť	F	T	F	F		1	ΓF	F	F			T
	F	F	Т	F	Т	Т		]	FF	Т	Т			F
	F	F	F	F	Т	F		]	ΓF	F	F			F

## c. Truth-functionally consistent

- E: Eugene O'Neill was an alcoholic.
- P: Eugene O'Neill's plays show that he was an alcoholic.
- I: The Iceman Cometh must have been written by a teetotaler.
- F: Eugene O'Neill was a fake.

				$\downarrow$	$\downarrow$	$\downarrow$		$\downarrow$	
E	F	Ι	Р	E	Р	Ι	Е	$\vee$	F
Т	Т	Т	Т	Т	Т	Т	Т	Т	Т
Т	Т	Т	F	Т	F	Т	Т	Т	Т
Т	Т	F	Т	Т	Т	F	Т	Т	Т
Т	Т	F	F	Т	F	F	Т	Т	Т
Т	F	Т	Т	Т	Т	Т	Т	Т	F
Т	F	Т	F	Т	F	Т	Т	Т	F
Т	F	F	Т	Т	Т	F	Т	Т	F
Т	F	F	F	Т	F	F	Т	Т	F
F	Т	Т	Т	F	Т	Т	F	Т	Т
F	Т	Т	F	F	F	Т	F	Т	Т
F	Т	F	Т	F	Т	F	F	Т	Т
F	Т	F	F	F	F	F	F	Т	Т
F	F	Т	Т	F	Т	Т	F	F	F
F	F	Т	F	F	F	Т	F	F	F
F	F	F	Т	F	Т	F	F	F	F
F	F	F	F	F	F	F	F	F	F

- e. Truth-functionally consistent
  - R: The Red Sox will win next Sunday.
  - J: Joan bet \$5.00.
  - E: Joan will buy Ed a hamburger.

				$\downarrow$					$\downarrow$	
Е	J	R	R	$\supset$	(J	$\supset$	E)	~ R	&	~ E
Т	Т	Т	Т	Т	Т	Т	Т	FΤ	F	FΤ
Т	Т	F	F	Т	Т	Т	Т	ΤF	F	FΤ
Т	F	Т	T	Т	F	Т	Т	FΤ	F	FΤ
Т	F	F	F	Т	F	Т	Т	ΤF	F	FΤ
F	Т	Т	T	F	Т	F	F	FΤ	F	ΤF
F	Т	F	F	Т	Т	F	F	ΤF	Т	ΤF
F	F	Т	Т	Т	F	Т	F	FΤ	F	ΤF
F	F	F	F	Т	F	Т	F	ΤF	Т	ΤF

**4.**a. First assume that  $\{\mathbf{P}\}$  is truth-functionally inconsistent. Then, since **P** is the only member of  $\{\mathbf{P}\}$ , there is no truth-value assignment on which **P** is true; so **P** is false on every truth-value assignment. But then ~ **P** is true on every truth-value assignment, and so ~ **P** is truth-functionally true.

Now assume that  $\sim \mathbf{P}$  is truth-functionally true. Then  $\sim \mathbf{P}$  is true on every truth-value assignment, and so  $\mathbf{P}$  is false on every truth-value assignment. But then there is no truth-value assignment on which  $\mathbf{P}$ , the only member of  $\{\mathbf{P}\}$ , is true, and so the set is truth-functionally inconsistent.

c. No. For example, 'A' and '~ A' are both truth-functionally indeterminate, but  $\{A, ~A\}$  is truth-functionally inconsistent.

### Section 3.5E

1.a. Truth-functionally valid

			А	Н	J	А	$\downarrow$ $\cap$	(H	&	J)		J	$\downarrow$	Н		↓ ~ J			↓ ~ A
			T T T T T F F F	T T F T T T	T F T F T F T F T	T T T T F F F	T F F T T T	T T F T T T F	T F F T F F	T F T F T F T		J T F T F T F T	T F F T T F F	T T F F T T F		FT FT FT FT FT FT FT			F T F T F T F T F T T F T F
А	D	c G	<b>F</b> . Tr   (D	F uth- ≡	F │ funct ~ G)	F tion ↓ &	T ally G	F valic	F I G ∨	<b>F</b>	Л	F D)	T &	<b>F</b> A])	→ ∩	<b>T F</b> ~ D	G	↓ ∩	<b>T F</b> ~ D
T T T T F F F F F	T F F T F F F	T F T F T F T F	T F F T F F	F T F F T F	F T T F F T T F F T T F F T T F	F F T F F T F	T F T F T F T F	ר ו ו ו ו ו ו ו ו ו ו ו ו ו ו ו ו ו ו ו	T T T T T T T T T T T T T T	T T T F F F F	T F F T T T	T F F T F F	T F F F F F F	T T T F F F F	F F T F T T T	F T F T T F F T F T T F T F	T F T F T F T F	F T T F T T T	F T F T T F T F F T F T T F

e. Truth-f	-functionally valid	
	, T T	
C D	$E \mid (C \supset D) \supset (D \supset E) \qquad D \qquad C \supset$	Е
тт	<u> </u>	Т
ТТ	F TTTFTFF T TF	F
ΤF	T TFFTFTTFTTFTT	Т
ΤF	F TFFTFTF F TF	F
F T	T FTTTTTT FT	Т
	r   r 1 1 r 1 r r 1 r 1 r 1 r 1 r 1 r 1	г Т
F F	F F F F F F F F F F F F F F F F F F F	F
or Truth 4	functionally valid	
g. mun-i		
G H	$\downarrow \qquad \qquad \downarrow \qquad \qquad \qquad \qquad \downarrow \qquad \qquad$	= H)
ТТ	TTTTFFFT FTTFTTFT TEETETETE	ТТ ББ
F T	FFT T TFT TFFFT TF	гг FT
FF	FTFTTFFF TFTTFTFF	TF
F G	$\downarrow \qquad \qquad \downarrow \qquad \qquad$	F
Т Т Т <b>F</b>	TFT T TFT FT T T T TFT F FTF TF FT F T	T T
F T	FTF T TFT FT TF TF	F
FF	FTF T FTF TFT FT	F
<b>2.</b> a. Truth-f	-functionally valid $\downarrow \qquad \qquad \downarrow \qquad \qquad \downarrow$ $\mid (J \lor M) \supset \sim (J \& M) \qquad M \equiv (M \supset J) \qquad M$	↓ ⊃ J
ТТ	ΤΤΤ Γ F F T T Τ Τ Τ Τ Τ Τ Τ Τ	ТТ
TF	TTFTTFF FFTT F	ТТ
FΤ	FTT TTFFT TF TFF T	FF
FF	FFFTTFFF FFFTF F	ΤF

с. Т	ruth-	funct	iona	lly va	alid											
			I							1				I.		
А	В	A	≁	~ A			(B	$\supset$	A)	≁	В		А	↓ ≡	~ B	
— Т	Т	Т	F	FТ			т	т	т	Т	Т		Т	F	FТ	-
T	F	T	F	FΤ			F	Т	Т	F	F		Т	Т	Τŀ	7
F	Т	F	Т	ΤF			Т	F	F	Т	Т		F	Т	FТ	
F	F	F	Т	ΤF			F	Т	F	F	F		F	F	Τŀ	7
е. Т	ruth-	funct	iona	lly in	nval	id										
٨	D (		↓ 0.	Γ/Φ	ρ.	$(\mathbf{C})$	_	(C	_	A ) 1	1	↓ ▶ _	D		C	↓ ⊃ C
	в с		~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~	~ [ ( <b>D</b>	~~~	C)		(C		A)]			~ D		· C .	
Т	FF	$\mathbf{T} \mid \mathbf{T}$	ΤŢ	ΓF	F	F	F	F	Т	Т	]	FΤ	ΤF	T	F	FF
<b>3.</b> a. T	ruth-	funct	tiona	lly v	alid											
D	C	/D	ρ.	$(\mathbf{C})$	↓	(D	.,	C								
D	C	(D	&	C)		(Б	~	C,	-							
Т	Т	T	Т	Т	Т	Т	Т	Т								
Т	F	T	F	F	Т	Т	Т	F								
F	Т	F F	F	Т	Т	F	Т	Т								
с. Т	ruth-	funct	iona	r llv ir	• nval	id	r	r								
				,									$\downarrow$			
J	Т	([(	J ⊃	T)	$\supset$	J]	&	[(]	Γ =	) J)	$\supset$	T])		(~ J	$\vee$	~ T)
Т	Т	] ]	ГТ	Т	Т	Т	Т	1	r 1	Т	Т	Т	F	FΤ	F	FΤ
е. Т	ruth-	funct	tiona	lly in	nval	id										
											$\downarrow$					
B	С	D	[(B	&	C)	&	(1	3 \	/ I	D)]	$\supset$	D				
Т	Т	F	Т	Т	Т	Т	]	ר ז	ΓI	7	F	F				
<b>4.</b> a. T	ruth-	funct	tiona	lly in	nval	id										
S	5: 'S	tern'	mea	uns ti	he s	sam	e as	s 'st	ar'.							
N	: 'N	lacht	' me	ans t	the	sam	ie a	s 'd	lav'.							
-					I.											
Ň	S	N	× ⊃	s -	~ N	* ~ ¦	S									
— T	Т	Т	Т	ТΙ	FΤ	F	Г									
T	F	T	F	FI	F T	T	F									
F	Т	F	Т	ΤŢ	ΓF	F	Г									
F	F	F	Т	F 7	ΓF	Т	F									

# c. Truth-functionally valid

- S: September has 30 days.
- A: April has 30 days.

N: November has 30 days.

F: February has 40 days.

M: May has 30 days.

						$\downarrow$								$\downarrow$				$\downarrow$
А	F	М	Ν	S	S	&	(A	&	N)	(A	. =	=	~ M)	&	(N	$\supset$	M)	F
Т	Т	Т	Т	Т	Т	Т	Т	Т	Т	Т	F	7	FΤ	F	Т	Т	Т	Т
Т	Т	Т	Т	F	F	F	Т	Т	Т	Т	F	7	FΤ	F	Т	Т	Т	Т
Т	Т	Т	F	Т	Т	F	Т	F	F	Т	F	7	FΤ	F	F	Т	Т	Т
Т	Т	Т	F	F	F	F	Т	F	F	Т	F	7	FΤ	F	F	Т	Т	Т
Т	Т	F	Т	Т	Т	Т	Т	Т	Т	Т	' 1	Г	ΤF	F	Т	F	F	Т
Т	Т	F	Т	F	F	F	Т	Т	Т	Т	' 1	Γ	ΤF	F	Т	F	F	Т
Т	Т	F	F	Т	Т	F	Т	F	F	Т	' ]	Г	ΤF	Т	F	Т	F	Т
Т	Т	F	F	F	F	F	Т	F	F	Т	' ]	Г	ΤF	Т	F	Т	F	Т
Т	F	Т	Т	Т	Т	Т	Т	Т	Т	Т	F	7	FΤ	F	Т	Т	Т	F
Т	F	Т	Т	F	F	F	Т	Т	Т	Т	F	ĩ	FΤ	F	Т	Т	Т	F
Т	F	Т	F	Т	Т	F	Т	F	F	Т	F	7	FΤ	F	F	Т	Т	F
Т	F	Т	F	F	F	F	Т	F	F	Т	F	ĩ	FΤ	F	F	Т	Т	F
Т	F	F	Т	Т	Т	Т	Т	Т	Т	Т	' 1	Г	ΤF	F	Т	F	F	F
Т	F	F	Т	F	F	F	Т	Т	Т	Т	' 1	Г	ΤF	F	Т	F	F	F
Т	F	F	F	Т	T	F	Т	F	F	Т	1	Г	ΤF	Т	F	Т	F	F
Т	F	F	F	F	F	F	Т	F	F	Т	1	Γ	ΤF	Т	F	Т	F	F
F	Т	Т	Т	Т	T	F	F	F	Т	F	' I	Г	FΤ	Т	Т	Т	Т	Т
F	Т	Т	Т	F	F	F	F	F	Т	F	' I	Γ	FΤ	Т	Т	Т	Т	Т
F	Т	Т	F	Т	T	F	F	F	F	F	' I	Γ	FΤ	Т	F	Т	Т	Т
F	Т	Т	F	F	F	F	F	F	F	F	' I	Γ	FΤ	Т	F	Т	Т	Т
F	Т	F	Т	Т	T	F	F	F	Т	F	F	7	ΤF	F	Т	F	F	Т
F	Т	F	Т	F	F	F	F	F	Т	F	F	ĩ	ΤF	F	Т	F	F	Т
F	Т	F	F	Т	T	F	F	F	F	F	F	7	ΤF	F	F	Т	F	Т
F	Т	F	F	F	F	F	F	F	F	F	F	ĩ	ΤF	F	F	Т	F	Т
F	F	Т	Т	Т	T	F	F	F	Т	F	' I	Γ	FΤ	Т	Т	Т	Т	F
F	F	Т	Т	F	F	F	F	F	Т	F	' T	Γ	FΤ	Т	Т	Т	Т	F
F	F	Т	F	Т	T	F	F	F	F	F	' I	Γ	FΤ	Т	F	Т	Т	F
F	F	Т	F	F	F	F	F	F	F	F	' I	Γ	FΤ	Т	F	Т	Т	F
F	F	F	Т	Т	T	F	F	F	Т	F	F	7	ΤF	F	Т	F	F	F
F	F	F	Т	F	F	F	F	F	Т	F	F	ĩ	ΤF	F	Т	F	F	F
F	F	F	F	Т	T	F	F	F	F	F	F	ĩ	ΤF	F	F	Т	F	F
F	F	F	F	F	F	F	F	F	F	F	F	ĩ	ΤF	F	F	Т	F	F

e. Truth-functionally valid

- D: Computers can have desires.
- E: Computers can have emotions.
- T: Computers can think.

				$\downarrow$			$\downarrow$			$\downarrow$		$\downarrow$
D	E	Т	T	=	Е	Ε	$\supset$	D	D	$\supset$	~ T	~ T
Т	Т	Т	Т	Т	Т	Т	Т	Т	Т	F	FΤ	FΤ
Т	Т	F	F	F	Т	Т	Т	Т	Т	Т	ΤF	ТБ
Т	F	Т	Т	F	F	F	Т	Т	Т	F	FΤ	FΤ
Т	F	F	F	Т	F	F	Т	Т	Т	Т	ΤF	ТБ
F	Т	Т	T	Т	Т	Т	F	F	F	Т	FΤ	FΤ
F	Т	F	F	F	Т	Т	F	F	F	Т	ΤF	ТБ
F	F	Т	T	F	F	F	Т	F	F	Т	FΤ	FΤ
F	F	F	F	Т	F	F	Т	F	F	Т	ΤF	ΤF

5.a. Suppose that the argument is truth-functionally valid. Then there is no truth-value assignment on which  $\mathbf{P}_1, \ldots, \mathbf{P}_n$  are all true and  $\mathbf{Q}$  is false. But, by the characteristic truth-table for '&', the iterated conjunction  $(\ldots (\mathbf{P}_1 \& \mathbf{P}_2) \& \ldots \mathbf{P}_n)$  has the truth-value  $\mathbf{T}$  on a truth-value assignment if and only if all of  $\mathbf{P}_1, \ldots, \mathbf{P}_n$  have the truth-value  $\mathbf{T}$  on that assignment. So, on our assumption, there is no truth-value assignment on which the antecedent of  $(\ldots (\mathbf{P}_1 \& \mathbf{P}_2) \& \ldots \& \mathbf{P}_n) \supset \mathbf{Q}$  has the truth-value  $\mathbf{T}$  and the consequent has the truth-value  $\mathbf{F}$ . It follows that there is no truth-value assignment on which the corresponding material conditional is false, so it is truth-functionally true.

Assume that  $(\ldots (\mathbf{P}_1 \& \mathbf{P}_2) \& \ldots \& \mathbf{P}_n) \supset \mathbf{Q}$  is truth-functionally true. Then there is no truth-value assignment on which the antecedent is true and the consequent false. But the iterated conjunction is true if and only if the sentences  $\mathbf{P}_1, \ldots, \mathbf{P}_n$  are all true. So there is no truth-value assignment on which  $\mathbf{P}_1, \ldots, \mathbf{P}_n$  are all true and  $\mathbf{Q}$  is false; hence the argument is truth-functionally valid.

c. No. For example,  $\{A \supset B\} \models `\sim A \lor B`$ . But  $\{A \supset B\}$  does not entail `~ A', nor does it entail `B'.

### Section 3.6E

**1.a.** If  $\{\sim \mathbf{P}\}$  is truth-functionally inconsistent, then there is no truth-value assignment on which  $\sim \mathbf{P}$  is true (since  $\sim \mathbf{P}$  is the only member of its unit set). But then  $\sim \mathbf{P}$  is false on every truth-value assignment, so  $\mathbf{P}$  is true on every truth-value assignment and is truth-functionally true.

c. If  $\Gamma \cup \{\sim \mathbf{P}\}$  is truth-functionally inconsistent, then there is no truthvalue assignment on which every member of  $\Gamma \cup \{\sim \mathbf{P}\}$  is true. But  $\sim \mathbf{P}$  is true on a truth-value assignment if and only if  $\mathbf{P}$  is false on that assignment. Hence there is no truth-value assignment on which every member of  $\Gamma$  is true and **P** is false. Hence  $\Gamma \models \mathbf{P}$ .

**2.a. P** is truth-functionally true if and only if the set {~ **P**} is truth-functionally inconsistent. But {~**P**} is the same set as  $\emptyset \cup$  {~ **P**}. So **P** is truth-functionally true if and only if  $\emptyset \cup$  {~ **P**} is truth-functionally inconsistent. But we have already seen, by previous results, that  $\emptyset \cup$  {~ **P**} is truth-functionally inconsistent if and only if  $\emptyset \models$  **P**. Hence **P** is truth-functionally true if and only if  $\emptyset \models$  **P**.

c. Assume that  $\Gamma$  is truth-functionally inconsistent. Then there is no truth-value assignment on which every member of  $\Gamma$  is true. Let **P** be an *arbitrarily* selected sentence of *SL*. Then there is no truth-value assignment on which every member of  $\Gamma$  is true and **P** false since there is no truth-value assignment on which every member of  $\Gamma$  is true. Hence  $\Gamma \models \mathbf{P}$ .

**3.**a. Let  $\Gamma$  be a truth-functionally consistent set. Then there is at least one truth-value assignment on which every member of  $\Gamma$  is true. But **P** is also true on such an assignment since a truth-functionally true sentence is true on every truth-value assignment. Hence on at least one truth-value assignment every member of  $\Gamma \cup \{\mathbf{P}\}$  is true; so the set is truth-functionally consistent.

**4.a. P** is either true or false on each truth-value assignment. On any assignment on which **P** is true, **Q** is true (because  $\{\mathbf{P}\} \models \mathbf{Q}$ ) and so  $\mathbf{Q} \lor \mathbf{R}$  is true. On any assignment on which **P** is false,  $\sim \mathbf{P}$  is true, **R** is therefore also true (because  $\{\sim \mathbf{P}\} \models \mathbf{R}$ ), and so  $\mathbf{Q} \lor \mathbf{R}$  is true as well. Either way, then,  $\mathbf{Q} \lor \mathbf{R}$  is true—so the sentence is truth-functionally true.

c. Assume that every member of  $\Gamma \cup \Gamma'$  is true on some truth-value assignment. Then every member of  $\Gamma$  is true, and so **P** is true (because  $\Gamma \models \mathbf{P}$ ). Every member of  $\Gamma'$  is also true, and so **Q** is true (because  $\Gamma' \models \mathbf{Q}$ ). Therefore **P** & **Q** is true. So  $\Gamma \cup \Gamma' \models \mathbf{P} \& \mathbf{Q}$ .