# PHILOSOPHY 220

Truth Functional Properties on Truth Trees

# THE SEMANTIC CONCEPTS OF TRUTH-FUNCTIONAL LOGIC:

- Tautology
- Contradiction
- Contingency
- Entailment
- Validity
- Equivalence
- Consistency

# CONSISTENCY ON TRUTH TREES

- A set of sentences of SL is consistent if and only if there is at least one truth value assignment [of the constituents of the set of sentences] on which all the members of the set are true.
- We now know how to check for consistency using a tree, and can recover specific truth-value assignments on which all members of a given set come out true (if the set is consistent).

# FLASHBACK !!!

# CONTRADICTION

- A sentence of SL is a contradiction if and only if it is false on every possible truth-value assignment of its constituents.

Definition

- A sentence **P** is truth-functionally false if and only if {**P**} is truth-functionally inconsistent.
- Since inconsistent sets are sets that can never all be true at the same time, and since the unit set of **P** has only one member, it must always be false to be inconsistent.

Explained via consistency

# SETTING UP A TREE TO CHECK FOR CONTRADICTION

- A sentence ℙ of SL is a contradiction if and only if {ℙ} has a closed truth tree (meaning {ℙ} is inconsistent).
- If the tree for {ℙ} closes, it means that it is impossible for ℙ to be true.

# TAUTOLOGY

- A sentence of SL is a tautology if and only if it is true on every possible truth-value assignment of its constituents.

**Definition**

- A sentence **P** is a tautology if and only if {~**P**} is truth-functionally inconsistent.
- The only member of any inconsistent set is a contradiction, and the negation of a contradiction is a tautology, so if ~**P** is a contradiction, then **P** is a tautology.

**Explained via Consistency**

# SETTING UP A TREE TO CHECK FOR TAUTOLOGY

- A sentence ℙ of SL is a tautology if and only if {~ℙ} has a closed truth tree (meaning {~ℙ} is inconsistent).
- If the tree for {~ℙ} closes, it means that it is impossible for ~ℙ to be true, which in turn means it is impossible for ℙ to be false.

# CONTINGENCY

- A sentence of SL is contingent if and only if it is neither a tautology nor a contradiction.

**Definition**

- A sentence **P** is truth-functionally indeterminate if and only if both {~**P**} and {**P**} are truth-functionally consistent.
- If the above are consistent, then **P** is neither a tautology nor a contradiction.

**Explained via consistency**

# SETTING UP A TREE TO CHECK FOR CONTINGENCY

- A sentence ℙ of SL is contingent if and only if neither {~ℙ} nor {ℙ} has a closed truth tree.
- If the tree for {~ℙ} is open and the tree for {ℙ} is open, then it means that ℙ can be either true or false.

# EQUIVALENCE

Definition

- Sentences P and Q of SL are equivalent if and only if there is no truth value assignment [for the components of P and Q] on which P and Q have different truth-values.

Explained via consistency

- Sentences P and Q of SL are equivalent if and only if {~(P ≡ Q)} is inconsistent
- If P and Q have the same truth values, P ≡ Q is a tautology. That would mean that ~(P ≡ Q) would be a contradiction, and so would make for an inconsistent set.

# SETTING UP A TREE TO CHECK FOR EQUIVALENCE

- Sentences $\mathbb{P}$ and $\mathbb{Q}$ are equivalent if and only if {~($\mathbb{P} \equiv \mathbb{Q}$)} has a closed truth tree.
- If $\mathbb{P}$ and $\mathbb{Q}$ are equivalent, then ($\mathbb{P} \equiv \mathbb{Q}$) is a tautology because equivalent sentences always have the same truth-value. That would make ~($\mathbb{P} \equiv \mathbb{Q}$) a contradiction.
- So to check for equivalence of any two sentences of SL on a tree, join them with a biconditional, negate the biconditional, and check for consistency of the set with that negated biconditional as its only member.

# ENTAILMENT

- A set $\Gamma$ of sentences of SL entails a sentence $\mathbb{P}$ if and only if there is no truth-value assignment on which every member of $\Gamma$ is true and $\mathbb{P}$ is false.

- $\Gamma \models \mathbb{P}$ if and only if $\Gamma \cup \{\sim\mathbb{P}\}$ is truth-functionally inconsistent.

Definition

Explained via Consistency

# ENTAILMENT ON A TREE

- A finite set $\Gamma$ entails a sentence $\mathbb{P}$ if and only if the set $\Gamma \cup \{\sim\mathbb{P}\}$ has a closed tree (is inconsistent).
- So to check if some finite set entails some sentence, represent each member of the set along with the negation of what you're checking to see whether the set entails.
- If the table closes, then it is impossible for all members of the set $\Gamma$ to be true while $\mathbb{P}$ is false.

# VALIDITY

- Since validity is simply a special case of entailment, the same procedure can demonstrate that validity can be described in terms of consistency.
- If an argument is valid, then the union of the set of its premises and the negation of its conclusion will form an inconsistent set.

# VALIDITY ON A TREE

- An argument with a finite number of premises is valid if and only if the set consisting of all and only its premises and the negation of the conclusion has a closed tree.
- A tree that closes when you include every premise and the negation of the conclusion means that the conclusion cannot be false while the premises are true.