

	1	$\sim(F \supset G)$	Given	
	2	$\sim(G \supset H)$	Given	Derive: I
1	3	$\sim I$	A1	$\sim I$ (contradiction)
2 1	4	G	A2	$\sim I$ (contradiction)
3 2 1	5	F	A3	$\supset I(G)$
3 2 1	6	G	4 R	
! 2 1	7	$F \supset G$	5-6 $\supset I$	
2 1	8	$(F \supset G) \ \& \ \sim(F \supset G)$	1,7 &I	contradiction
! 1	9	$\sim G$	4-8 $\sim I$	Note: $\sim H \supset \sim G$ is equivalent to $G \supset H$
4 1	10	$\sim H$	A4	$\supset I(\sim G)$
4 1	11	$\sim G$	9 R	
! 1	12	$\sim H \supset \sim G$	10-11 $\supset I$	See note on 9
5 1	13	G	A5	$\supset I(H)$
6 5 1	14	$\sim H$	A6	$\sim I$ (contradiction)
6 5 1	15	$\sim G$	12,14 $\supset E$	
6 5 1	16	$G \ \& \ \sim G$	13,15 &I	contradiction
! 5 1	17	$\sim\sim H$	14-16 $\sim I$	
5 1	18	H	17 $\sim E$	
! 1	19	$G \supset H$	13-18 $\supset I$	
1	20	$(G \supset H) \ \& \ \sim(G \supset H)$	2,19 &I	Contradiction
!	21	$\sim\sim I$	3-20 $\sim I$	
	22	I	21 $\sim E$	QED

Alternate (If you want the proof to be harder than it has to be):

	1	$\sim(F \supset G)$	Given	
	2	$\sim(G \supset H)$	Given	Derive: I
1	3	$\sim(G \vee \sim G)$	A1	$\sim I$ (contradiction)
2 1	4	G	A2	$\sim I$ (contradiction)
2 1	5	$G \vee \sim G$	4 vI	
2 1	6	$(G \vee \sim G) \& \sim(G \vee \sim G)$	3,5 &I	contradiction
! 1	7	$\sim G$	4-6 $\sim I$	
1	8	$G \vee \sim G$	7 vI	
1	9	$(G \vee \sim G) \& \sim(G \vee \sim G)$	3,8 &I	contradiction
!	10	$\sim\sim(G \vee \sim G)$	3-9 $\sim I$	
	11	$G \vee \sim G$	10 $\sim E$	
1a	12	G	A1a	vE (I)
3 1a	13	$\sim I$	A3	$\sim I$ (contradiction)
4 3 1a	14	F	A4	$\supset I$ (G)
4 3 1a	15	G	12 R	
! 3 1a	16	$F \supset G$	14-15 $\supset I$	
3 1a	17	$(F \supset G) \& \sim(F \supset G)$	1,16 &I	contradiction
! 1a	18	$\sim\sim I$	13-17 $\sim I$	
1a	19	I	18 $\sim E$	
1b	20	$\sim G$	A2b	vE (I)
5 1b	21	$\sim I$	A5	$\sim I$ (contradiction)
6 5 1b	22	$\sim H$	A6	$\supset I$ ($\sim G$)
6 5 1b	23	$\sim G$	20 R	
! 5 1b	24	$\sim H \supset \sim G$	22-23 $\supset I$	
7 5 1b	25	G	A7	$\supset I$ (H)
8 7 5 1b	26	$\sim H$	A8	$\sim I$ (contradiction)
8 7 5 1b	27	$\sim G$	24,26 $\supset E$	
8 7 5 1b	28	$G \& \sim G$	25,27 &I	contradiction
! 7 5 1b	29	$\sim\sim H$	26-29 $\sim I$	
7 5 1b	30	H	29 $\sim E$	
! 5 1b	31	$G \supset H$	25-30 $\supset I$	
5 1b	32	$(G \supset H) \& \sim(G \supset H)$	2,31 &I	contradiction
! 1b	33	$\sim\sim I$	21-32 $\sim I$	
1b	34	I	33 $\sim E$	
!	35	I	11,12-19,20-34 vE	QED

