Derivations 1

PHILOSOPHY 220

SYNTACTIC METHODS

- Truth tables and truth trees are both designed to find a set of truth-value assignments on which certain things are the case. That makes trees and tables *semantic* methods for finding truth-functional properties.
- We can also construct a system that looks only at the structure of sentences of SL to determine truth-functional properties. This system is a syntactic system.

PROOFS, DERIVATIONS, DEDUCTIONS

- The terminology of this text does not always reflect actual common usage.
- What this text calls ‘derivations’ are also called ‘proofs’, ‘deductions’, or ‘deductive proofs’.

THE PURPOSE OF DERIVATION

- The derivation system allows us to start with certain assumptions, and see if we may arrive at some conclusion.
- If we can, we have a chain of reasoning written down that proves that a given claim follows from a given set of assumptions.

TWO KINDS OF ASSUMPTIONS

- For derivations, we will have two kinds of assumptions:
  - The set that we start with
  - Other assumptions that we make along the way (these assumptions introduce **subderivations**)
- The text formats these using vertical and horizontal lines that are difficult to reproduce on a standard word processor. So what follows is an alternate format.

ALTERNATE DERIVATION FORMAT (168)

The numerals to the left of the line numerals are called dependency numerals. They indicate that what is on that line depends on the assumption so enumerated.

Derive G ⊃ (H ⊃ K)

	1.	(G & H) ⊃ K	A
1	**2.**	G	A1/⊃I
2 1	**3.**	H	A2/⊃I
2 1	**4.**	G & H	2,3 &I
2 1	**5.**	K	1,4 ⊃E
!1	**6.**	H ⊃ K	3-5 ⊃I
!	**7.**	G ⊃ (H ⊃ K)	2-6 ⊃I

RULES THAT DO NOT REQUIRE ADDITIONAL ASSUMPTIONS:

- Conjunction Elimination &E
- Conjunction Introduction &I
- Negation Elimination ~E
- Disjunction Introduction vI
- Conditional Elimination ⊃E
- Biconditional Elimination ≡E
- Reiteration R

CONJUNCTION INTRODUCTION (&I)

L. P

M. Q

N. P & Q (or Q & P)   L, M &I

CONJUNCTION ELIMINATION (&E)

M. P & Q

N. P (or Q)   M &E

CONJUNCTION RULES

NEGATION ELIMINATION (~E)

- Note: This is a different rule than listed in the text (the text makes no effective distinction between their ~I and their ~E)!!

M. ~~P

N. P M ~E

DISJUNCTION INTRODUCTION ($\vee$I)

M. P

N. P v Q (or Q v P)   M vI

CONDITIONAL ELIMINATION (⊃E)

L. $P \supset Q$

M. P

N. Q L,M ⊃E

Notice that this is really just good old fashioned Modus Ponens.

BICONDITIONAL ELIMINATION (≡E)

M. $P \equiv Q$

N. $(P \supset Q) \,\&\, (Q \supset P)$ M ≡E

Note that this also is a different rule than the text supplies for biconditional elimination.

REITERATION

N. P

M. P N, R

(when no additional assumptions are involved)

PRACTICE:

5.1.1, 1d. Derive $(D \& E) \& (\sim B \& C)$

1. $\sim B \supset (D \& E)$ A
2. $(A \& \sim B) \& C$ A

What first?

PRACTICE:

5.1.1, 1d. Derive (D & E) & (~B & C)

1. ~B ⊃ (D & E) A
2. (A & ~B) & C A

Since the main connective is ‘&’, we know that our last step will be &I, which means we need each conjunct on its own line.

PRACTICE:

5.1.1, 1d. Derive (D & E) & (~B & C)

1. ~B ⊃ (D & E) A

2. (A & ~B) & C A

We can straightforwardly get the second conjunct from applications of &E and &I to line 2.

PRACTICE:

5.1.1, 1d. Derive (D & E) & (~B & C)

1. ~B ⊃ (D & E)	A
2. (A & ~B) & C	A
3. A & ~B	2, &E

What now?

PRACTICE:

5.1.1, 1d. Derive (D & E) & (~B & C)

1. ~B ⊃ (D & E)	A
2. (A & ~B) & C	A
3. A & ~B	2, &E
4. ~B	3, &E

What now?

PRACTICE:

5.1.1, 1d. Derive (D & E) & (~B & C)

1. ~B ⊃ (D & E)	A
2. (A & ~B) & C	A
3. A & ~B	2, &E
4. ~B	3, &E
5. C	2, &E

What now?

PRACTICE:

5.1.1, 1d. Derive (D & E) & (~B & C)

1. ~B ⊃ (D & E)	A
2. (A & ~B) & C	A
3. A & ~B	2, &E
4. ~B	3, &E
5. C	2, &E
6. ~B & C	4,5 &I

What now?

PRACTICE:

5.1.1, 1d. Derive (D & E) & (~B & C)

1. ~B ⊃ (D & E)	A
2. (A & ~B) & C	A
3. A & ~B	2, &E
4. ~B	3, &E
5. C	2, &E
6. ~B & C	4,5 &I

Now we need the other conjunct.

PRACTICE:

5.1.1, 1d.	Derive (D & E) & (~B & C)
1. ~B ⊃ (D & E)	A
2. (A & ~B) & C	A
3. A & ~B	2, &E
4. ~B	3, &E
5. C	2, &E
6. ~B & C	4,5 &I

See it here as this consequent?

PRACTICE:

5.1.1, 1d. Derive (D & E) & (~B & C)

1. ~B ⊃ (D & E)	A
2. (A & ~B) & C	A
3. A & ~B	2, &E
4. ~B	3, &E
5. C	2, &E
6. ~B & C	4,5 &I

We happen to have the antecedent too!

PRACTICE:

5.1.1, 1d. Derive (D & E) & (~B & C)

1. ~B ⊃ (D & E)	A
2. (A & ~B) & C	A
3. A & ~B	2, &E
4. ~B	3, &E
5. C	2, &E
6. ~B & C	4,5 &I
7. (D & E)	1,4 ⊃E

Now what?

PRACTICE:

5.1.1, 1d. Derive (D & E) & (~B & C)

1. ~B ⊃ (D & E)	A
2. (A & ~B) & C	A
3. A & ~B	2, &E
4. ~B	3, &E
5. C	2, &E
6. ~B & C	4,5 &I
7. (D & E)	1,4 ⊃E
8. (D & E) & (~B & C)	6,7 &I

QED