Philosophy 104

Chapter 6 Notes (Part 2)

The Language of Logic

- To analyze the logical structure of our language, we need to strip away its content. That is the purpose of symbolic logic.
- The elements of symbolic logic so far:
  - Propositional variables (p, q, r, …)
  - Connectives (~, &, v)
- We will now introduce punctuation:
  - [Brackets] and (parentheses)

Syntax rules for Symbolic Logic

1. A single propositional variable is a wff (well-formed formula)
2. If P is a wff, then ~P is a wff (The negation applies to the wff to its immediate right)
   - One and only one wff is in the scope of the negation symbol
3. If P and Q are wffs then P v Q and P & Q are wffs (conjunctions and disjunctions connect wffs)
   - Each wff in a conjunction or disjunction is in the scope of the conjunction or disjunction symbol
4. Any wff may be in the scope of no more than one connective. (use parentheses to indicate scope when necessary)

Parentheses

- Consider:
  - p & q & r
- Is this a wff?

Parentheses

- Consider:
  - p & q v r
- This is not a wff because the q is in the scope of more than one connective.

Parentheses

- Consider:
  - p & q v r
- This is not a wff because the q is in the scope of more than one connective.
- This can be fixed with parentheses:
  - (p & q) v r
  - p & (q v r)

Parentheses

- Consider:
  - p & q v r
- This is not a wff because the q is in the scope of more than one connective.
- This can be fixed with parentheses:
  - (p & q) v r -- Either p and q, or r
  - p & (q v r) -- p and either q or r

Parentheses

- (p & q) v r
  - This is a disjunction that has a conjunction as one of its disjuncts.
  - The disjunction symbol is the main connective because the entire expression is in its scope.

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  - This is a disjunction that has a conjunction as one of its disjuncts.
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- p & (q v r)
  - This is a conjunction that has a disjunction as one of its conjuncts.
  - The conjunction symbol is the main connective because the entire expression is in its scope.

Parentheses

- (p & q) v r
  - This is a disjunction that has a conjunction as one of its disjuncts.
  - The disjunction symbol is the main connective because the entire expression is in its scope.
- p & (q v r)
  - This is a conjunction that has a disjunction as one of its conjuncts.
  - The conjunction symbol is the main connective because the entire expression is in its scope.
- Since each is a wff, [(p & q) v r] & [p & (q v r)] is a wff.

PARENTHESES

- Consider:
  - ~a & g
  - ~(a & g)

PARENTHESES

- Consider:
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- Substitute "Annie is rich" for 'a' and "Gina is happy" for 'g'.

PARENTHESES

- Consider:
  - ~a & g
  - ~(a & g)
- Substitute “Annie is rich” for ‘a’ and “Gina is happy” for ‘g’.
- The first phrase translates to “It is not the case that Annie is rich and it is the case that Gina is happy.”
- The second phrase translates to “It is not the case that both Annie is rich and Gina is happy.”

Logic and Math

- I know that logic LOOKS for all the world like math, but resist the temptation to treat mathematical symbols and logical symbols as interchangeable.

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- I know that logic LOOKS for all the world like math, but resist the temptation to treat mathematical symbols and logical symbols as interchangeable.
- For example, math has parentheses, and also has a negative symbol, "-" that looks a bit like logic's negation symbol "~", so since changing –(2 + 3) to -2 + -3 is a mathematically valid procedure, changing ~(p & q) to ~p & ~q should be logically valid, right?

NO!!!

- ~(p & q) means "not both", meaning that p might be true or q might be true, but not both at the same time.
- ~p & ~q means that p is false and q is false.

The two are clearly different. Moral of the story: Logical symbols may resemble, but are not mathematical symbols.

Brackets and Parentheses

- Brackets and Parentheses are interchangeable, they are just visually distinct.

EXERCISE XII

10. ~[A v ~(Z v X)]
    ~[T v ~(F v F)]
    ~[T v ~(F)]
    ~[T v T]
    ~[T]
    F

11. ~A v ~(Z v X)
    ~T v ~(F v F)
    F v ~(F v F)
    F v ~(F)
    F v T
    T

Procedure for using truth tables to find validity:

1. Create the reference columns (one per propositional variable, in alpha. order)
2. Create one column for each logical connective (v,&,~, ⊃)
3. Fill in reference columns:
   a) # of rows = 2^n where n = # of propositional variables
   b) Fill first half of leftmost column's rows with value 'T', the rest with value 'F'
   c) Fill next column with T and F, beginning with T and having exactly half as many consecutive iterations of T and F as occurs in column leftward.
   d) Repeat (c) until reference columns are filled in.
4. Fill in the rest of the table
5. Check for any case in which the premises are all true and the conclusion is false. If such a case is on the table, the argument form is invalid, valid otherwise.