PHILOSOPHY 200

Symbolization in SL

Symbolic Language *SL*

- In order to make it easier to examine the structure of our language and arguments, logicians have worked out a way of substituting symbols for the various parts of sentences in natural language. The language that results from this abstraction process is a symbolic language that we call *SL.*
- (SL stands for “Sentential Logic”)

The basic units of *SL*

- The sentence is the basic unit of the language *SL*.

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  - Put another way, we will be concerned with sentences that express single propositions; that is, sentences that refer to one state of affairs that is or is not the case.
  - These sentences are called **simple sentences**
  - Examples of simple sentences:
    - The cat is on the mat.
    - California borders Oregon.
    - Biff owns a car.
    - Buffalo buffalo buffalo.

Compound sentences

- The concept of a compound sentence in logic is similar to the concept of a compound sentence in English grammar, but not exactly the same.
  - For example, “I am wearing a red hat and red shoes” is not a grammatically compound sentence because ‘red shoes’ is not, by itself, a sentence.
  - It is, however, a compound sentence in logic because it expresses two propositions (*I am wearing a red hat; I am wearing red shoes)* and joins them together (*and*).
- A compound sentence in logic is one that expresses more than a single proposition, joined together in some way.

Truth-Functionality

- To say that the truth of some compound sentence is a function of the truth of its parts and the way in which they are connected is to say that that compound sentence is joined with a truth-functional connective.
- Consider:
  - “I’m Bob, and I’m a Libra”
  - This sentence expresses two propositions: *I’m Bob; I’m a Libra.*
  - The propositions expressed are joined by the word ‘*and’.*
  - The sentence “I’m Bob, and I’m a Libra” is true if the speaker is indeed Bob, *and* if the speaker is indeed a Libra.

A Truth Table:

Each component of the sentence:		The sentence:
I'm Bob	I'm a Libra	I'm Bob and I'm a Libra
T	T	T
T	F	F
F	T	F
F	F	F

Another Truth Table:

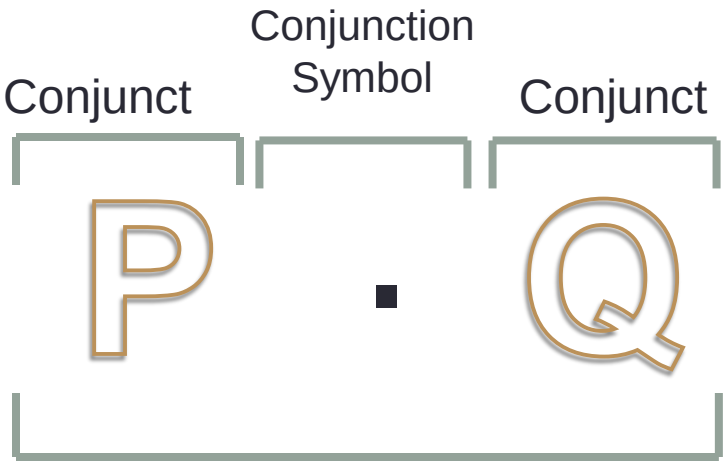
Each component of the sentence:		The sentence:
I'm tired	I'm hungry	I'm tired or hungry
T	T	T
T	F	T
F	T	T
F	F	F

The vocabulary of *SL*

- Every simple sentence can be represented in *SL* as a capital letter .
- The choice of capital letter is arbitrary.
- Single capital letters that stand for simple sentences are called **atomic sentences.**
- Any compound sentence can be represented by connecting atomic sentences by means of truth-functional connectives.
- Examples:
  - “I’m Bob” in *SL* can be “B”

The vocabulary of *SL*

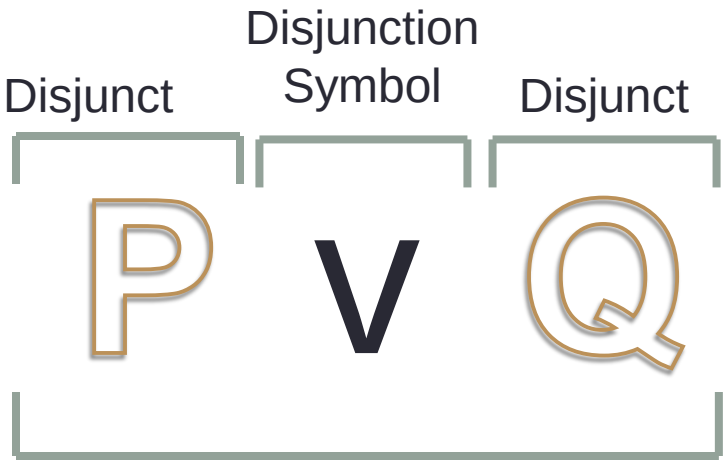
- Truth-functional connectives also have symbols in *SL*.



Conjunction

Conjunction Translation

- Contrast:
  - “Galen and Watson are physicians”
- With
  - “The priest married John and Cathy”
- The first is an instance of truth-functional conjunction, symbolizable as G · W (Galen is a physician and Watson is a physician)
- The second is not an instance of truth-functional conjunction. The sentence does not mean that the priest married John and also that the priest married Cathy. Rather, the priest married John and Cathy to one another. This is symbolizable as M.



Disjunction

Disjunction Translation

- Contrast:
  - “I rode my bicycle yesterday or the day before.”
- With:
  - “You may have either a hamburger or a hot dog”
- The first is an example of an inclusive ‘or’, which translates straightforwardly to the ‘v’, yielding Y v T.
- The second is an example of the exclusive ‘or’ which yields (H v D) · ~(H · D), or “You may have a hamburger or a hot dog and not both a hamburger and a hot dog”.

Negation Symbol

Negation

Negation Translation

- If B is “My cat has black fur”, then what is an acceptable English translation of ‘~B’?
  - My cat does not have fur.
  - My cat has white fur.
  - The cat that is not mine has black fur
  - I do not have a cat that has black fur.
  - My dog has black fur.
  - It is not the case that my cat has black fur
- Pay special attention to negations of English sentences containing ‘all’, ‘no’, and ‘some’

Scope

- When we use connectives to join atomic sentences of *SL*, we must be concerned with the **scope** of the connectives we use. Parentheses () and Brackets [] help us to visually organize scope for molecular sentences in *SL*.
- Contrast
  - ~(A · B): It is not the case that both A and B
  - ~A · B: Both A is not the case and B is the case.
- The difference between the above is that the entire molecular sentence ‘A · B’ is in the scope of the negation, while in the sentence ‘~A · B’, only ‘A’ is in the scope of the negation.

Logic is not math!!!

- While '~' certainly looks like '-', and while 'negation' and 'negative' sound like they ought to have a great deal to do with one another, resist the temptation to treat the logical negation symbol like the mathematical negative symbol.
- Example:
  - Does -(3 + 5) = -3 + -5?
  - Is ~(A · B) truth-functionally equivalent to ~A · ~B?

- Let's Check:

Equivalence on a Truth Table

Ref.		First Sent.			Second Sent.		
A	B	~	A · B		~A	·	~B
T	T	**F**	T		F	**F**	F
T	F	**T**	F		F	**F**	T
F	T	**T**	F		T	**F**	F
F	F	**T**	F		T	**T**	T

‘~ (A · B)’ is not logically equivalent to ‘~A · ~B’ because they do not have the same truth values in the same circumstances.

Truth Functionality Illustrated:

- Consider the molecular sentence :
- (A · B) v [(~B v A) · (~A v B)]
- Now assume A is true and B is false. What is the truth value of the whole sentence?

(A · B) v [(~B v A) · (~A v B)]

(T · F) v [(~F v T) · (~T v F)]

F v [(~F v T) · (~T v F)]

F v [(T v T) · (F v F)]

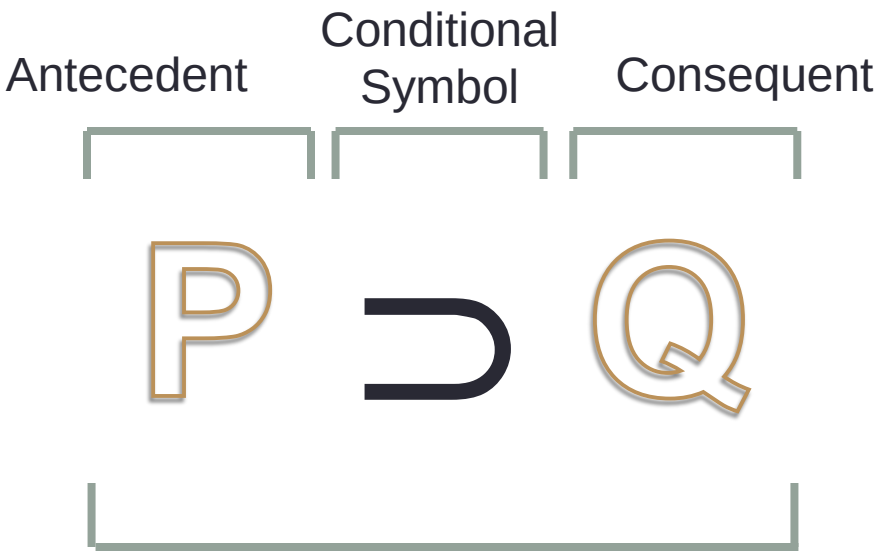
F v [T · (F v F)]

F v [T · F]

F v F

F

The Material Conditional



Material Conditional Definition

P	Q	P ⊃ Q
T	T	T
T	F	
F	T	
F	F	

Very Straightforward. “If you clean the barn then I pay you five bucks.” is true when it is true that you clean the barn and when it is true that I pay you five bucks.

Material Conditional Definition

P	Q	$P \supset Q$
T	T	T
T	F	F
F	T	
F	F	

Also Straightforward. “If you clean the barn then I pay you five bucks.” is false when it is true that you clean the barn and false that I pay you five bucks.

Material Conditional Definition

P	Q	$P \supset Q$
T	T	T
T	F	F
F	T	T
F	F	T

A bit counterintuitive: “If you clean the barn then I pay you five bucks.” is true whenever it is not false. If the antecedent is false (you do not clean out the barn) then the conditional will not be falsified, and will be counted as true. Whether I give you five bucks or not, I still haven’t lied to you.

Material Conditional Equivalence

- Consider whether the following are logically equivalent:
  - “If you clean the barn I’ll pay you $5.”
  - “Either you don’t clean the barn, or I’ll pay you $5”
- The preceding are symbolized:
  - $C \supset P$
  - $\sim C \vee P$

Material Conditional Equivalence

P	Q	~P	v	Q
T	T	F	**T**	T
T	F	F	**F**	F
F	T	T	**T**	T
F	F	T	**T**	F

Material Conditional Equivalence

P	Q	~P	v	Q	P ⊃ Q
T	T	F	**T**	T	**T**
T	F	F	**F**	F	**F**
F	T	T	**T**	T	**T**
F	F	T	**T**	F	**T**

Material Conditional Equivalence

- Many students want to make a conditional false when the antecedent is false. That would make the symbol '⊃' mean the same thing as the '·'.
- Does 'If P then Q' mean the same thing as 'P and Q'?
- Clearly not. The person who utters the latter is asserting the truth of both P and Q while the person who utters the former is asserting neither the truth nor falsity of either P or Q.
- The material conditional asserts a relationship between P and Q that is false when the antecedent (P) is true while the consequent (Q) is false, and true otherwise.

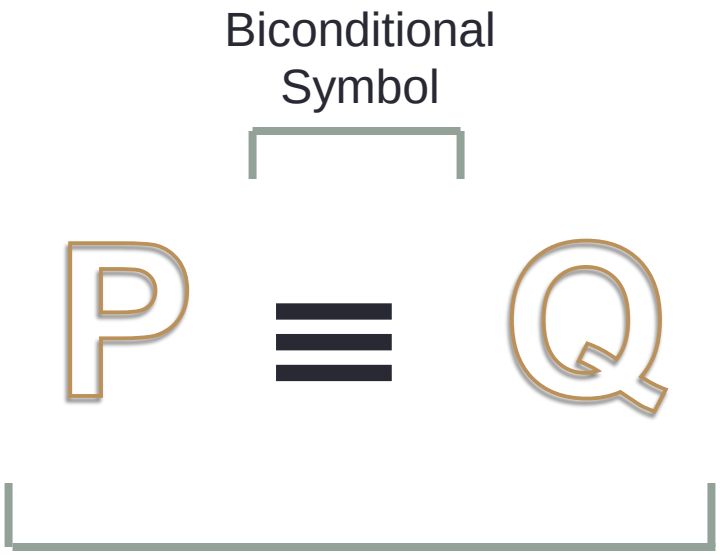
Material Conditional and the English 'If…Then…'

- Many uses of "If…Then…" in English are not instances of the material conditional.
- Consider the truth value of: "If there is an Elephant in the room, then it is raining."
  - The above is true (barring an elephant being in the room and clear weather when I present these notes)
  - If you think it must be false, you are reading it as a causal conditional, which is a material conditional with extra baggage. In a causal conditional "If P then Q" means "P causes Q"

Material Conditional and the English 'If…Then…'

- Many uses of "If…Then…" in English are not instances of the material conditional.
- Consider symbolizing: "If the Germans had won the second world war, then everyone would speak German"
  - Notice that there are not *two* propositions expressed because 'the Germans *had* won…' does not express a proposition by itself, nor does 'everyone *would* speak German'.
  - This is a counterfactual, or subjunctive conditional. It is best symbolized 'P'.

The Material Biconditional



Material Biconditional Definition

P	Q	P ≡ Q
T	T	T
T	F	F
F	T	F
F	F	T

Material Biconditional and '='

- The biconditional is a sign of logical equivalence and not general equivalence or identity.
- The sentence 'P $\supset$ Q' is logically equivalent to the sentence '~P v Q' but is not *the same sentence*.
- So '(P $\supset$ Q) $\equiv$ (~P v Q)' is necessarily true while

  '(P $\supset$ Q) = (~P v Q)' is false