Philosophy 200

proof

The concept of proof

- To prove something is to give a valid argument for its being the case.

- To prove that an argument is valid is to give a valid argument that demonstrates the argument's validity.

- A proof then, is a specific kind of valid argument.

- When you assume only the premises and can derive the conclusion from them, you have demonstrated that an argument is valid.

The format of a proof

- A proof of validity starts with assuming all of the premises and only the premises, and writing each on its own line.

- Every line of a proof is something that is at least assumed to be true, and every line of a proof is assumed true for a reason.

- That reason is called the justification and is also written on each line.

Example:

1. P ⊃ Q   Premise

2. P   Premise

← Justifications

Lines

Implication

- One good reason to assume something is true is that it is logically implied by other things that are assumed true.

- The most basic kinds of valid argument forms are the tools we use justify some steps of a proof.
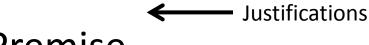
- And here they are:

Modus Ponens

- Any time a line of a proof is a conditional, and a different line of a proof is the antecedent of that conditional, then a further line of the proof can be the consequent of that conditional.

Example:

1. P ⊃ Q   Premise

                     ← Justifications

2. P   Premise

↑

Lines

Example:

1. P ⊃ Q   Premise
2. P   Premise   ← Justifications
3. Q

↑
Lines

Example:

1. P ⊃ Q   Premise

2. P   Premise ← Justifications

3. Q   1,2, MP

↑

Lines

Modus Tollens

- Whenever one line of a proof is a conditional and another line of a proof is the negation of the consequent of that conditional, a further line may be the negation of the antecedent.

Example:

1. P ⊃ (Q v R)   Premise
2. ~(Q v R)   Premise   ⟵ Justifications

↑
Lines

Example:

1. P ⊃ (Q v R)   Premise
2. ~(Q v R)   Premise
3. ~P

Justifications

Lines

Example:

1. P ⊃ (Q v R)   Premise
2. ~(Q v R)   Premise   ← Justifications
3. ~P   1,2, MT

↑
Lines

Hypothetical Syllogism

- When the consequent of a conditional in a proof is the same as the antecedent of a different conditional in a proof, then the antecedent of the former conditional and the consequent of the latter conditional may be on a different line of the proof as antecedent and consequent (respectively) of another conditional.

Extended Example

1. If the Bull-Moose candidate is elected, then a new tax bill will pass.   Premise

2. If a new tax bill passes, then the economy will crash.   Premise

Extended Example

1. If the Bull-Moose candidate is elected, then a new tax bill will pass.

   Premise

2. If a new tax bill passes, then the economy will crash.

   Premise

3. If the Bull-Moose candidate is elected, then the economy will crash

Extended Example

1. If the Bull-Moose candidate is elected, then a new tax bill will pass.

   Premise

2. If a new tax bill passes, then the economy will crash.

   Premise

3. If the Bull-Moose candidate is elected, then the economy will crash

   1,2, HS

Extended Example

1. B ⊃ T   Premise
2. T ⊃ E   Premise
3. B ⊃ E   1,2, HS

Disjunctive Syljogism

- This reasoning is often known as the process of elimination.

- Whenever a line of a proof is a disjunction and another line is the negation of one of those disjuncts, a further line may assert the truth of the other disjunct.

Example

1. (P v Q) v ~R   Premise
2. R   Premise

Example

1. (P v Q) v ~R   Premise
2. R   Premise
3. P v Q

Example

1. (P v Q) v ~R   Premise
2. R   Premise
3. P v Q   1,2, DS

A proof and what it means

#	Sentence of SL	Justification	(Explanation)
1	~R	Premise	
2	P ⊃ S	Premise	
3	R v ~S	Premise	
4	~P ⊃ Q	Premise	
5			
6			
7			

A proof and what it means

#	Sentence of SL	Justification	(Explanation)
1	~R	Premise	Premises are assumed to be true for the sake of proving that the conclusion can be derived from them. In this case, the conclusion will be the sentence 'Q'.
2	P ⊃ S	Premise	
3	R v ~S	Premise	
4	~P ⊃ Q	Premise	
5			
6			
7			

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3	R v ~S	Premise	
4	~P ⊃ Q	Premise	
5	~S		
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2	P ⊃ S	Premise	
3	R v ~S	Premise	
4	~P ⊃ Q	Premise	
5	~S	1,3, DS	
6			
7			

A proof and what it means

#	Sentence of SL	Justification	(Explanation)
1	~R	Premise	Premises are assumed to be true for the sake of proving that the conclusion can be derived from them. In this case, the conclusion will be the sentence 'Q'.
2	P ⊃ S	Premise	
3	R v ~S	Premise	
4	~P ⊃ Q	Premise	
5	~S	1,3, DS	Assumed true because it is a valid implication.
6			
7			

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6	~P		
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2	P ⊃ S	Premise	
3	R v ~S	Premise	
4	~P ⊃ Q	Premise	
5	~S	1,3, DS	Assumed true because it is a valid implication.
6	~P	2,5, MT	Assumed true because it is a valid implication.
7			

A proof and what it means

#	Sentence of SL	Justification	(Explanation)
1	~R	Premise	Premises are assumed to be true for the sake of proving that the conclusion can be derived from them. In this case, the conclusion will be the sentence 'Q'.
2	P ⊃ S	Premise	
3	R v ~S	Premise	
4	~P ⊃ Q	Premise	
5	~S	1,3, DS	Assumed true because it is a valid implication.
6	~P	2,5, MT	Assumed true because it is a valid implication.
7	Q		

A proof and what it means

#	Sentence of SL	Justification	(Explanation)
1	~R	Premise	Premises are assumed to be true for the sake of proving that the conclusion can be derived from them. In this case, the conclusion will be the sentence 'Q'.
2	P ⊃ S	Premise	
3	R v ~S	Premise	
4	~P ⊃ Q	Premise	
5	~S	1,3, DS	Assumed true because it is a valid implication.
6	~P	2,5, MT	Assumed true because it is a valid implication.
7	Q	4,6, MP	Assumed true because it is a valid implication.

A proof and what it means

#	Sentence of SL	Justification	(Explanation)
1	~R	Premise	Premises are assumed to be true for the sake of proving that the conclusion can be derived from them. In this case, the conclusion will be the sentence 'Q'.
2	P ⊃ S	Premise	
3	R v ~S	Premise	
4	~P ⊃ Q	Premise	
5	~S	1,3, DS	Assumed true because it is a valid implication.
6	~P	2,5, MT	Assumed true because it is a valid implication.
7	Q	4,6, MP	Assumed true because it is a valid implication.

QED. The proof is finished and it demonstrates that the conclusion, Q, logically follows from the premises listed above.

Proof is a skill

- Understanding a proof when it is done for you is one thing.

- Being able to do it yourself is another.

- Getting competent with proofs is a skill, not simply a body of knowledge. Simply memorizing the rules doesn't tell you when to use which rule or why.

An analogy

- Knowing how to add or subtract does not mean knowing when to use addition or subtraction to figure something out.

- Knowing how to subtract but not practicing it means working slowly and making mistakes while subtracting.

- The same things apply, *mutatis mutandis*, for logical proof.