Philosophy 200

Indirect truth tables

Indirect truth tables

- You may have noticed that only certain entries in a truth table are ultimately of interest to us.

- We can exploit that fact by constructing only the parts of a truth table that we need.

- This can save a great deal of time and paper, but it requires a perfect understanding of what is going on.

Testing for contingence

- If a sentence of SL is contingent, then it is possible for that sentence to be true or to be false, so only two rows of a truth table are required.

- You should assume that the sentence in question is true, and assume it is false, and see if a coherent set of truth assignments is possible for each.

Testing for contingence

- If a sentence of SL is contingent, then it is possible for that sentence to be true or to be false, so only two rows of a truth table are required.

- You should assume that the sentence in question is true, and assume it is false, and see if a coherent set of truth assignments is possible for each.

- If it is, then the sentence is contingent.

Tautology/Contradiction

- If there is no coherent row on which a sentence is true, it is a contradiction.

- If there is no coherent row on which a sentence is false, it is a tautology.

Equivalence

- As with tautology and contradiction, we test for equivalence by looking for a counterexample.
- If we assume that one of the sentences is true and the other false, then either we will or will not get a coherent truth-value assignment. If we do, then the two sentences are shown not to be equivalent. If we cannot get a coherent truth-value assignment assuming that one sentence is true while the other is false, then we must try it the other way before drawing any conclusions. (why?)

~(B · ~A) and (A v B)

A	B	~	(B ·	~A)		A v B

Which columns should be identical if these two sentences are equivalent?

~(B · ~A) and (A v B)

A	B	~	(B ·	~A)		A v B

Which columns should be identical if these two sentences are truth-functionally equivalent?

~(B · ~A) and (A v B)

A	B	~	(B ·	~A)		A v B
		T				F

So assume that one is T and the other F.

~(B · ~A) and (A v B)

A	B	~	(B ·	~A)		A v B
		T				F

Note that the only way for A v B to be false is for both A and B to be false.

~(B · ~A) and (A v B)

A	B	~	(B ·	~A)		A v B
F	F	T				F

Note that the only way for A v B to be false is for both A and B to be false.

~(B · ~A) and (A v B)

A	B	~	(B ·	~A)		A v B
F	F	T		T		F

Now we see if this truth-assignment is coherent...

~(B · ~A) and (A v B)

A	B	~	(B ·	~A)		A v B
F	F	T	F	T		F

It is coherent. If A and B are both false, then ~(B · ~A) must be true.

F · (J v H)   and   (F · J) v H

F	H	J	F ·	(J v H)		(F · J)	v H

Which columns should be identical if these two sentences are truth-functionally equivalent?

F · (J v H)   and   (F · J) v H

F	H	J	F ·	(J v H)		(F · J)	v H

Which columns should be identical if these two sentences are truth-functionally equivalent?

F · (J v H)   and   (F · J) v H

F	H	J	F ·	(J v H)		(F · J)	v H
			T				F

Assume one is T and the other F…

F · (J v H)   and   (F · J) v H

F	H	J	F ·	(J v H)		(F · J)	v H
	F		T			F	F

If the disjunction '(F · J) v H' is false, then both disjuncts must be false.

F · (J v H)   and   (F · J) v H

F	H	J	F ·	(J v H)		(F · J)	v H
T	F		T	T		F	F

If the conjunction 'F · (J v H)' is true then both of its conjuncts must be true.

F · (J v H)   and   (F · J) v H

F	H	J	F ·	(J v H)		(F · J)	v H
T	F	?	T	T		F	F

Is there a truth-value assignment for J that is coherent?

F · (J v H)   and   (F · J) v H

F	H	J	F ·	(J v H)		(F · J)	v H
T	F	T	T	T		F	F

FAIL!

Is there a truth-value assignment for J that is coherent?

T is not coherent because it would make (F · J) true, which would
in turn make ((F · J) v H) true.

F · (J v H)   and   (F · J) v H

F	H	J	F ·	(J v H)			(F · J)	v H
T	F	F	T	T			F	F

FAIL!

Is there a truth-value assignment for J that is coherent?

F is not coherent because it would make (J v H) false, which would in turn make (F · (J v H)) false.

F · (J v H)   and   (F · J) v H

F	H	J	F ·	(J v H)		(F · J)	v H
T	F	X	T	T		F	F

FAIL!

Does this mean that F · (J v H) and (F · J) v H are equivalent?

F · (J v H)   and   (F · J) v H

F	H	J	F ·	(J v H)		(F · J)	v H
T	F	X	T	T		F	F

FAIL!

Does this mean that F · (J v H) and (F · J) v H are equivalent?

IT DOES NOT!

We have shown that F · (J v H) cannot be true while (F · J) v H is false, but it is still possible that F · (J v H) can be false while (F · J) v H is true.

So let's check:

F · (J v H)   and   (F · J) v H

F	H	J	F ·	(J v H)		(F · J)	v H
			F				T

So assume one is F and the other T (the opposite of what we began with)…

F · (J v H)   and   (F · J) v H

F	H	J	F ·	(J v H)		(F · J)	v H
F	T		F				T

We have many ways to proceed here, so let us assume that F is false (to make the conjunction it is in false) and that H is true (to make the disjunction that it is in true). If this turns out to be incoherent, there are several other possibilities to try.

F · (J v H)   and   (F · J) v H

F	H	J	F ·	(J v H)		(F · J)	v H
F	T	?	F				T

Now we must see if any truth value of J would yield a coherent table…

F · (J v H)   and   (F · J) v H

F	H	J	F ·	(J v H)		(F · J)	v H
F	T	T	F				T

Let's try T first.

F · (J v H)　and　(F · J) v H

F	H	J	F ·	(J v H)		(F · J)	v H
F	T	T	F				T

'J v H' comes out true on this set of assignments while 'F · J' comes out false.

F · (J v H)   and   (F · J) v H

F	H	J	F ·	(J v H)		(F · J)	v H
F	T	T	F	T		F	T

This is a coherent truth-value assignment for F, H, and J that reveals that these two sentences are not truth-functionally equivalent.

Consistence

- Since consistence means that each member of the set can be true at the same time, we can prove coherence on a truth table by showing what happens on a row in which all members of the set are true.

- If there is a coherent set of truth-assignments, we prove consistence. If there is not, we prove inconsistence.

Validity

- Validity requires there never to be any row on which the premises are true while the conclusion is false, so that is the row we try to build to check validity.

- If there is a coherent set of truth assignments for that row, the argument is invalid. If there is not, the argument is valid.