Section 9.1E

a.	1.	$(\exists x)Fx$ ✓	SM
	2.	$(\exists x) \sim Fx$ ✓	SM
	3.	Fa	1 $\exists$D
	4.	$\sim Fb$	2 $\exists$D
		o	

The tree has a completed open branch.

c.	1.	$(\exists x)(Fx \ \& \sim Gx)$ ✓		SM
	2.	$(\forall x)(Fx \supset Gx)$		SM
	3.	$Fa \ \& \sim Ga$ ✓		1 $\exists$D
	4.	Fa		3 &D
	5.	$\sim Ga$		3 &D
	6.	$Fa \supset Ga$ ✓		2 $\forall$D
	7.	$\sim Fa$	Ga	6 $\supset$D
		$\times$	$\times$	

The tree is closed.

e.	1.	$\sim (\forall x)(Fx \supset Gx)$ ✓	SM
	2.	$\sim (\exists x)Fx$ ✓	SM
	3.	$\sim (\exists x)Gx$ ✓	SM
	4.	$(\exists x) \sim (Fx \supset Gx)$ ✓	1 $\sim \forall$D
	5.	$(\forall x) \sim Fx$	2 $\sim \exists$D
	6.	$(\forall x) \sim Gx$	3 $\sim \exists$D
	7.	$\sim (Fa \supset Ga)$ ✓	4 $\exists$D
	8.	Fa	7 $\sim \supset$D
	9.	$\sim Ga$	7 $\sim \supset$D
	10.	$\sim Fa$	5 $\forall$D
		$\times$	

The tree is closed.

g.	1.	$(\exists x)Fx$ ✓	SM
	2.	$(\exists y)Gy$ ✓	SM
	3.	$(\exists z)(Fz \ \& \ Gz)$ ✓	SM
	4.	Fa	1 $\exists$D
	5.	Gb	2 $\exists$D
	6.	$Fc \ \& \ Gc$ ✓	3 $\exists$D
	7.	Fc	6 &D
	8.	Gc	6 &D
		o	

The tree has a completed open branch.

i.

1.	$(\forall x)(\forall y)(Fxy \supset Fyx)$		SM
2.	$(\exists x)(\exists y)(Fxy \,\&\, \sim Fyx)$ ✓		SM
3.	$(\exists y)(Fay \,\&\, \sim Fya)$ ✓		2 $\exists$D
4.	$Fab \,\&\, \sim Fba$ ✓		3 $\exists$D
5.	Fab		4 &D
6.	$\sim Fba$		4 &D
7.	$(\forall y)(Fay \supset Fya)$		1 $\forall$D
8.	$Fab \supset Fba$ ✓		7 $\forall$D
9.	$\sim Fab$	Fba	8 $\supset$D
	×	×	

The tree is closed.

k.

1.	$(\exists x)Fx \supset (\forall x)Fx$ ✓		SM
2.	$\sim (\forall x)(Fx \supset (\forall y)Fy)$ ✓		SM
3.	$(\exists x) \sim (Fx \supset (\forall y)Fy)$ ✓		2 $\sim \forall$D
4.	$\sim (Fa \supset (\forall y)Fy)$ ✓		3 $\exists$D
5.	Fa		4 $\sim \supset$D
6.	$\sim (\forall y)Fy$ ✓		4 $\sim \supset$D
7.	$(\exists y) \sim Fy$ ✓		6 $\sim \forall$D
8.	$\sim Fb$		7 $\exists$D
9.	$\sim (\exists x)Fx$ ✓	$(\forall x)Fx$	1 $\supset$D
10.	$(\forall x) \sim Fx$		9 $\sim \exists$D
11.	$\sim Fa$		10 $\forall$D
12.	×	Fb	9 $\forall$D
		×	

The tree is closed.

m.

1.	$(\forall x)(Fx \supset (\exists y)Gyx)$		SM
2.	$\sim (\forall x) \sim Fx$ ✓		SM
3.	$(\forall x)(\forall y) \sim Gxy$		SM
4.	$(\exists x) \sim \sim Fx$ ✓		2 $\sim \forall$D
5.	$\sim \sim Fa$ ✓		4 $\exists$D
6.	Fa		5 $\sim \sim$ D
7.	$Fa \supset (\exists y)Gya$ ✓		1 $\forall$D
8.	$\sim Fa$	$(\exists y)Gya$ ✓	7 $\supset$D
9.	×	Gba	8 $\exists$D
10.		$(\forall y) \sim Gby$	3 $\forall$D
11.		$\sim Gba$	10 $\forall$D
		×	

The tree is closed.

o. 1.	$(\exists x)Lxx$ ✓		SM
2.	$\sim (\exists x)(\exists y)(Lxy \,\&\, Lyx)$ ✓		SM
3.	$(\forall x) \sim (\exists y)(Lxy \,\&\, Lyx)$		$2 \sim \exists D$
4.	Laa		$1\ \exists D$
5.	$\sim (\exists y)(Lay \,\&\, Lya)$ ✓		$3\ \forall D$
6.	$(\forall y) \sim (Lay \,\&\, Lya)$		$5 \sim \exists D$
7.	$\sim (Laa \,\&\, Laa)$ ✓		$6\ \forall D$
8.	$\sim Laa$	$\sim Laa$	$7 \sim \&D$
	×	×	

The tree is closed.

q. 1.	$(\exists x)(Fx \vee Gx)$ ✓		SM
2.	$(\forall x)(Fx \supset \sim Gx)$		SM
3.	$(\forall x)(Gx \supset \sim Fx)$		SM
4.	$\sim (\exists x)(\sim Fx \vee \sim Gx)$ ✓		SM
5.	$(\forall x) \sim (\sim Fx \vee \sim Gx)$		$4 \sim \exists D$
6.	$Fa \vee Ga$		$1\ \exists D$
7.	$Fa \supset \sim Ga$ ✓		$2\ \forall D$
8.	$Ga \supset \sim Fa$ ✓		$3\ \forall D$
9.	$\sim (\sim Fa \vee \sim Ga)$ ✓		$5\ \forall D$
10.	$\sim \sim Fa$ ✓		$9 \sim \vee D$
11.	$\sim \sim Ga$ ✓		$9 \sim \vee D$
12.	Ga		$11 \sim \sim D$
13.	Fa		$10 \sim \sim D$
14.	$\sim Fa$	$\sim Ga$	$7 \supset D$
	×	×	

The tree is closed.

Section 9.2E

Note: In these answers, whenever a tree is open we give a complete tree. This is because the strategies we have suggested do not uniquely determine the order of decomposition, and so the first open branch to be completed on your tree may not be the first such branch completed on our tree. In accordance with strategy 5, you should stop when your tree has one completed open branch.

a. 1.	$(\forall x)Fx \vee (\exists y)Gy$ ✓		SM
2.	$(\exists x)(Fx \;\&\; Gb)$ ✓		SM
3.	$Fa \;\&\; Gb$ ✓		2 $\exists$D
4.	Fa		3 &D
5.	Gb		3 &D
6.	$(\forall x)Fx$	$(\exists y)Gy$ ✓	1 $\vee$D
7.	Fa		6 $\forall$D
8.	Fb		6 $\forall$D
9.	o	Gc	6 $\exists$D
		o	

The tree has two completed open branches. The set is quantificationally consistent.

c. 1.	$(\forall x)(Fx \supset Gxa)$		SM
2.	$(\exists x)Fx$ ✓		SM
3.	$(\forall y) \sim Gya$		SM
4.	Fb		2 $\exists$D
5.	$Fb \supset Gba$ ✓		1 $\forall$D
6.	$\sim Fb$	Gba	5 $\supset$D
7.	$\times$	$\sim Gba$	3 $\forall$D
		$\times$	

The tree is closed. The set is quantificationally inconsistent.

e. 1.	$(\forall x)(Fx \supset Gxa)$			SM
2.	$(\exists x)Fx$ ✓			SM
3.	$(\forall y)Gya$			SM
4.	Fb			2 $\exists$D
5.	$Fb \supset Gba$ ✓			1 $\forall$D
6.	$\sim Fb$	Gba		5 $\supset$D
7.	$\times$	Gaa		3 $\forall$D
8.		$Fa \supset Gaa$ ✓		1 $\forall$D
9.		$\sim Fa$	Gaa	8 $\supset$D
		o	o	

The tree has two completed open branches. The set is quantificationally consistent.

The literals ‘Fb’, ‘Gba’, ‘Gaa’, and ‘~ Fa’ on the left completed open branch will all be true on any interpretation that makes the following assignments:

UD:	{1, 2}
a:	1
b:	2
Fx:	x is even
Gxy:	x is greater than or equal to y

The literals ‘Fb’, ‘Gba’, ‘Gaa’ on the right completed open branch will also be true on any interpretation that makes these assignments.

g. 1.	$(\forall x)(Fx \vee Gx)$		SM
2.	$\sim(\exists y)(Fy \vee Gy)$ ✓		SM
3.	$(\forall y) \sim (Fy \vee Gy)$		$2 \sim \exists D$
4.	$\sim(Fa \vee Ga)$ ✓		$3\ \forall D$
5.	$\sim Fa$		$4 \sim \vee D$
6.	$\sim Ga$		$4 \sim \vee D$
7.	$Fa \vee Ga$ ✓		$1\ \forall D$
8.	Fa	Ga	$7\ \vee D$
	×	×	

The tree is closed. The set is quantificationally inconsistent.

i. 1.	$(\forall z)Hz$		SM
2.	$(\exists x)Hx \supset (\forall y)Fy$ ✓		SM
3.	Ha		$1\ \forall D$
4.	$\sim(\exists x)Hx$ ✓	$(\forall y)Fy$	$2 \supset D$
5.	$(\forall x) \sim Hx$		$4 \sim \exists D$
6.	$\sim Ha$		$5\ \forall D$
7.	×	Fa	$4\ \forall D$
		o	

The tree has one completed open branch. The set is quantificationally consistent.

The literals ‘Ha’ and ‘Fa’ on the completed open branch will both be true on any interpretation that makes the following assignments:

UD:	{1}
a:	1
Fx:	x is a positive integer
Hx:	x is odd

k.

1.	$(\forall x)(\forall y)Lxy$		SM
2.	$(\exists z) \sim Lza \supset (\forall z) \sim Lza \checkmark$		SM
3.	$(\forall y)Lay$		1 $\forall$D
4.	Laa		3 $\forall$D
5.	$\sim (\exists z) \sim Lza \checkmark$	$(\forall z) \sim Lza$	2 $\supset$D
6.		$\sim Laa$	5 $\forall$D
7.	$(\forall z) \sim \sim Lza$	$\times$	5 $\sim \exists$D
8.	$\sim \sim Laa \checkmark$		7 $\forall$D
9.	Laa		8 $\sim \sim$ D
	o		

The tree has one completed open branch. The set is quantificationally consistent.

The literal ‘Laa’ on the completed open branch will be true on any interpretation that makes the following assignments:

UD: {1}
Lxy: x is less than or equal to y

m.

1.	$(\forall x)(Rx \equiv \sim Hxa)$					SM
2.	$\sim (\forall y) \sim Hby \checkmark$					SM
3.	Ra					SM
4.	$(\exists y) \sim \sim Hby \checkmark$					2 $\sim \forall$D
5.	$\sim \sim Hbc \checkmark$					4 $\exists$D
6.	Hbc					5 $\sim \sim$ D
7.	$Ra \equiv \sim Haa \checkmark$					1 $\forall$D
8.	$Rb \equiv \sim Hba \checkmark$					1 $\forall$D
9.	$Rc \equiv \sim Hca \checkmark$					1 $\forall$D
10.	Ra				$\sim Ra$	7 $\equiv$D
11.	$\sim Haa$				$\sim \sim Haa$	7 $\equiv$D
					$\times$	
12.	Rb		$\sim Rb$			8 $\equiv$D
13.	$\sim Hba$		$\sim \sim Hba \checkmark$			8 $\equiv$D
14.	Rc	$\sim Rc$	Rc	$\sim Rc$		9 $\equiv$D
15.	$\sim Hca$	$\sim \sim Hca \checkmark$	$\sim Hca$	$\sim \sim Hca \checkmark$		9 $\equiv$D
16.	o	Hca		Hca		15 $\sim \sim$ D
17.		o	Hba	Hba		13 $\sim \sim$ D
			o	o		

The tree has four completed open branches (the leftmost four). The set is quantificationally consistent.

The literals ‘Ra’, ‘Rb’, ‘Rc’, ‘Hbc’, ‘~ Haa’, ‘~ Hba’, and ‘~ Hca’ on the left-most completed open branch will all be true on any interpretation that makes the following assignments:

UD: {1, 2, 3}
a: 3
b: 1
c: 2
Fx: x is a positive integer
Hxy: 2 times x is equal to y

The literals ‘Ra’, ‘Rb’, ‘Rc’, ‘Hbc’, ‘~ Haa’, ‘~ Hba’, and ‘~ Hca’ on the second completed open branch will all be true on any interpretation that makes the following assignments:

UD: {1, 2, 3}
a: 1
b: 2
c: 3
Rx: x is less than 3
Hxy: x + y is greater than 3

The literals ‘Ra’, ‘Rb’, ‘Rc’, ‘Hbc’, ‘~ Haa’, ‘~ Hba’, and ‘~ Hca’ on the third completed open branch will all be true on any interpretation that makes the following assignments:

UD: {1, 2, 3}
a: 1
b: 3
c: 2
Rx: x is less than 3
Hxy: x + y is greater than 3

The literals ‘Ra’, ‘Rb’, ‘Rc’, ‘Hbc’, ‘~ Haa’, ‘~ Hba’, and ‘~ Hca’ on the fourth completed open branch will all be true on any interpretation that makes the following assignments:

UD: {1, 2, 3}
a: 1
b: 2
c: 3
Rx: x is less than 2
Hxy: x + y is greater than 2

Section 9.3E

1.a.

1.	$\sim ((\exists x)Fx \vee \sim (\exists x)Fx)$ ✓	SM
2.	$\sim (\exists x)Fx$ ✓	$1 \sim \vee D$
3.	$\sim \sim (\exists x)Fx$ ✓	$1 \sim \vee D$
4.	$(\forall x) \sim Fx$	$2 \sim \exists D$
5.	$(\exists x)Fx$ ✓	$3 \sim \sim D$
6.	Fa	$5\ \exists D$
7.	$\sim Fa$	$4\ \forall D$
	$\times$	

The tree is closed. The sentence ‘$(\exists x)Fx \vee \sim (\exists x)Fx$’ is quantificationally true.

c.

1.	$\sim ((\forall x)Fx \vee (\forall x) \sim Fx)$ ✓	SM
2.	$\sim (\forall x)Fx$ ✓	$1 \sim \vee D$
3.	$(\exists x) \sim Fx$ ✓	$1 \sim \vee D$
4.	$(\exists x) \sim Fx$ ✓	$2 \sim \forall D$
5.	$(\exists x) \sim \sim Fx$ ✓	$3 \sim \forall D$
6.	$\sim Fa$	$4\ \exists D$
7.	$\sim \sim Fb$ ✓	$5\ \exists D$
8.	Fb	$7 \sim \sim D$

The tree has a completed open branch, therefore the given sentence is not quantificationally true.

e.

1.	$\sim ((\forall x)Fx \vee (\exists x) \sim Fx)$ ✓	SM
2.	$\sim (\forall x)Fx$ ✓	$1 \sim \vee D$
3.	$\sim (\exists x) \sim Fx$ ✓	$1 \sim \vee D$
4.	$(\exists x) \sim Fx$ ✓	$2 \sim \forall D$
5.	$(\forall x) \sim \sim Fx$	$3 \sim \exists D$
6.	$\sim Fa$	$4\ \exists D$
7.	$\sim \sim Fa$ ✓	$5\ \forall D$
8.	Fa	$7 \sim \sim D$
	$\times$	

The tree is closed. The sentence ‘$(\forall x)Fx \vee (\exists x) \sim Fx$’ is quantificationally true.

g.

1.	$\sim ((\forall x)(Fx \vee Gx) \supset ((\exists x) \sim Fx \supset (\exists x)Gx))$ ✓		SM
2.	$(\forall x)(Fx \vee Gx)$		$1 \sim \supset D$
3.	$\sim ((\exists x) \sim Fx \supset (\exists x)Gx)$ ✓		$1 \sim \supset D$
4.	$(\exists x) \sim Fx$ ✓		$3 \sim \supset D$
5.	$\sim (\exists x)Gx$ ✓		$3 \sim \supset D$
6.	$(\forall x) \sim Gx$		$5 \sim \exists D$
7.	$\sim Fa$		$4\ \exists D$
8.	$Fa \vee Ga$ ✓		$2\ \forall D$
9.	Fa	Ga	$8 \vee D$
10.	$\times$	$\sim Ga$	$6\ \forall D$
		$\times$	

The tree is closed. The sentence ‘$(\forall x)(Fx \vee Gx) \supset [(\exists x) \sim Fx \supset (\exists x)Gx]$’ is quantificationally true.

i. 1.	~ (((∀x)Fx ∨ (∀x)Gx) ⊃ (∀x)(Fx ∨ Gx))✓		SM
2.	(∀x)Fx ∨ (∀x)Gx✓		1 ~ ⊃D
3.	~ (∀x)(Fx ∨ Gx)✓		1 ~ ⊃D
4.	(∃x) ~ (Fx ∨ Gx)✓		3 ~ ∀D
5.	~ (Fa ∨ Ga)✓		4 ∃D
6.	~ Fa		5 ~ ∨D
7.	~ Ga		5 ~ ∨D
8.	(∀x)Fx	(∀x)Gx	2 ∨D
9.	Fa	Ga	8 ∀D
	×	×	

The tree is closed. The sentence '((∀x)Fx ∨ (∀x)Gx) ⊃ (∀x)(Fx ∨ Gx)' is quantificationally true.

k. 1.	~ ((∃x)(Fx & Gx) ⊃ ((∃x)Fx & (∃x)Gx))✓		SM
2.	(∃x)(Fx & Gx)✓		1 ~ ⊃D
3.	~ ((∃x)Fx & (∃x)Gx)✓		1 ~ ⊃D
4.	Fa & Ga✓		2 ∃D
5.	Fa		4 &D
6.	Ga		4 &D
7.	~ (∃x)Fx✓	~ (∃x)Gx✓	3 ~ &D
8.	(∀x) ~ Fx	(∀x) ~ Gx	7 ~ ∃D
9.	~ Fa	~ Ga	8 ∀D
	×	×	

The tree is closed. The sentence '(∃x)(Fx & Gx) ⊃ ((∃x)Fx & (∃x)Gx)' is quantificationally true.

m. 1.	~ (~ (∃x)Fx ∨ (∀x) ~ Fx)✓	SM
2.	~ ~ (∃x)Fx✓	1 ~ ∨D
3.	~ (∀x) ~ Fx✓	1 ~ ∨D
4.	(∃x)Fx✓	2 ~ ~ D
5.	(∃x) ~ ~ Fx✓	3 ~ ∀D
6.	Fa	4 ∃D
7.	~ ~ Fb✓	5 ∃D
8.	Fb	7 ~ ~ D
	o	

The tree has a completed open branch, therefore the given sentence is not quantificationally true.

o.	1.	~ ((∀x)((Fx & Gx) ⊃ Hx) ⊃ (∀x)(Fx ⊃ (Gx & Hx)))✓				SM
	2.	(∀x)((Fx & Gx) ⊃ Hx)				1 ~ ⊃D
	3.	~ (∀x)(Fx ⊃ (Gx & Hx))✓				1 ~ ⊃D
	4.	(∃x) ~ (Fx ⊃ (Gx & Hx))✓				3 ~ ∀D
	5.	~ (Fa ⊃ (Ga & Ha))✓				4 ∃D
	6.	Fa				5 ~ ⊃D
	7.	~ (Ga & Ha)✓				5 ~ ⊃D
	8.	(Fa & Ga) ⊃ Ha✓				2 ∀D
	9.	~ (Fa & Ga)✓		Ha		8 ⊃D
	10.	~ Ga	~ Ha	~ Ga o	~ Ha ×	7 ~ &D
	11.	~ Fa × ~ Ga o	~ Fa × ~ Ga o			9 ~ &D

The tree has at least one completed open branch, therefore the given sentence is not quantificationally true.

q.	1.	~ ((∀x)(Fx ⊃ Gx) ⊃ (∀x)(Fx ⊃ (∀y)Gy))✓		SM
	2.	(∀x)(Fx ⊃ Gx)		1 ~ ⊃D
	3.	~ (∀x)(Fx ⊃ (∀y)Gy)✓		1 ~ ⊃D
	4.	(∃x) ~ (Fx ⊃ (∀y)Gy)✓		3 ~ ∀D
	5.	~ (Fa ⊃ (∀y)Gy)✓		4 ∃D
	6.	Fa		5 ~ ⊃D
	7.	~ (∀y)Gy✓		5 ~ ⊃D
	8.	(∃y) ~ Gy✓		7 ~ ∀D
	9.	~ Gb		8 ∃D
	10.	Fa ⊃ Ga✓		2 ∀D
	11.	~ Fa	Ga	10 ⊃D
	12.	×	Fb ⊃ Gb✓	2 ∀D
	13.		~ Fb o Gb ×	12 ⊃D

The tree has a completed open branch, therefore the given sentence is not quantificationally true.

s. 1.	$\sim ((\forall x)Gxx \supset (\forall x)(\forall y)Gxy)\checkmark$	SM
2.	$(\forall x)Gxx$	$1 \sim \supset D$
3.	$\sim (\forall x)(\forall y)Gxy\checkmark$	$1 \sim \supset D$
4.	$(\exists x) \sim (\forall y)Gxy\checkmark$	$3 \sim \forall D$
5.	$\sim (\forall y)Gay\checkmark$	$4\ \exists D$
6.	$(\exists y) \sim Gay\checkmark$	$5 \sim \forall D$
7.	$\sim Gab$	$6\ \exists D$
8.	Gaa	$2\ \forall D$
9.	Gbb	$2\ \forall D$
	o	

The tree has a completed open branch, therefore the given sentence is not quantificationally true.

u. 1.	$\sim ((\exists x)(\forall y)Gxy \supset (\forall x)(\exists y)Gyx)\checkmark$	SM
2.	$(\exists x)(\forall y)Gxy\checkmark$	$1 \sim \supset D$
3.	$\sim (\forall x)(\exists y)Gyx\checkmark$	$1 \sim \supset D$
4.	$(\exists x) \sim (\exists y)Gyx\checkmark$	$3 \sim \forall D$
5.	$(\forall y)Gay$	$2\ \exists D$
6.	$\sim (\exists y)Gyb\checkmark$	$4\ \exists D$
7.	$(\forall y) \sim Gyb$	$6 \sim \exists D$
8.	Gab	$5\ \forall D$
9.	$\sim Gab$	$7\ \forall D$
	×	

The tree is closed. The sentence '$(\exists x)(\forall y)Gxy \supset (\forall x)(\exists y)Gyx$' is quantificationally true.

w. 1.	$\sim (((\exists x)Lxx \supset (\forall y)Lyy) \supset (Laa \supset Lgg))\checkmark$		SM
2.	$(\exists x)Lxx \supset (\forall y)Lyy\checkmark$		$1 \sim \supset D$
3.	$\sim (Laa \supset Lgg)\checkmark$		$1 \sim \supset D$
4.	Laa		$3 \sim \supset D$
5.	$\sim Lgg$		$3 \sim \supset D$
6.	$\sim (\exists x)Lxx\checkmark$	$(\forall y)Lyy$	$2 \supset D$
7.	$(\forall x) \sim Lxx$		$6 \sim \exists D$
8.	$\sim Laa$		$7\ \forall D$
9.	×	Lgg	$6\ \forall D$
		×	

The tree is closed. The sentence '$[(\exists x)Lxx \supset (\forall y)Lyy] \supset (Laa \supset Lgg)$' is quantificationally true.

2.a.

1.	$(\forall x)Fx \ \& \ (\exists x) \sim Fx$ ✓	SM
2.	$(\forall x)Fx$	1 &D
3.	$(\exists x) \sim Fx$ ✓	1 &D
4.	$\sim Fa$	3 $\exists$D
5.	Fa	2 $\forall$D
	×	

c.

1.	$(\exists x)Fx \ \& \ (\exists x) \sim Fx$ ✓	SM
2.	$(\exists x)Fx$ ✓	1 &D
3.	$(\exists x) \sim Fx$ ✓	1 &D
4.	Fa	2 $\exists$D
5.	$\sim Fb$	3 $\exists$D
	o	

The tree has at least one completed open branch. Therefore, the given sentence is not quantificationally false.

e.

1.	$(\forall x)(Fx \supset (\forall y) \sim Fy)$		SM
2.	$Fa \supset (\forall y) \sim Fy$ ✓		1 $\forall$D
3.	$\sim Fa$	$(\forall y) \sim Fy$	2 $\supset$D
4.	o	$\sim Fa$	3 $\forall$D
		o	

The tree has at least one completed open branch. Therefore, the given sentence is not quantificationally false.

g.

1.	$(\forall x)(Fx \equiv \sim Fx)$		SM
2.	$Fa \equiv \sim Fa$ ✓		1 $\forall$D
3.	Fa	$\sim Fa$	2 $\equiv$D
4.	$\sim Fa$	$\sim \sim Fa$ ✓	2 $\equiv$D
5.	×	Fa	4 $\sim\sim$D
		×	

The tree is closed. Therefore the sentence is quantificationally false.

i.

1.	$(\exists x)(\exists y)(Fxy \ \& \sim Fyx)$ ✓	SM
2.	$(\exists y)(Fay \ \& \sim Fya)$ ✓	1 $\exists$D
3.	$Fab \ \& \sim Fba$ ✓	2 $\exists$D
4.	Fab	3 &D
5.	$\sim Fba$	3 &D
	o	

The tree has a completed open branch. Therefore, the given sentence is not quantificationally false.

k. 1.	$(\forall x)(\forall y)(Fxy \supset \sim Fyx)$		SM
2.	$(\forall y)(Fay \supset \sim Fya)$		1 $\forall$D
3.	$Faa \supset \sim Faa$ ✓		2 $\forall$D
4.	$\sim Faa$	$\sim Faa$	3 $\supset$D
	o	o	

The tree has at least one completed open branch. Therefore, the given sentence is not quantificationally false.

m. 1.	$(\exists x)(\forall y)Gxy \;\&\; \sim (\forall y)(\exists x)Gxy$ ✓	SM
2.	$(\exists x)(\forall y)Gxy$ ✓	1 &D
3.	$\sim (\forall y)(\exists x)Gxy$ ✓	1 &D
4.	$(\exists y) \sim (\exists x)Gxy$ ✓	3 $\sim \forall$D
5.	$(\forall y)Gay$	2 $\exists$D
6.	$\sim (\exists x)Gxb$ ✓	4 $\exists$D
7.	$(\forall x) \sim Gxb$	6 $\sim \exists$D
8.	Gab	5 $\forall$D
9.	$\sim Gab$	7 $\forall$D
	$\times$	

The tree is closed. Therefore the sentence is quantificationally false.

3.a. 1.	$\sim ((\exists x)Fxx \supset (\exists x)(\exists y)Fxy)$ ✓	SM
2.	$(\exists x)Fxx$ ✓	1 $\sim \supset$D
3.	$\sim (\exists x)(\exists y)Fxy$ ✓	1 $\sim \supset$D
4.	$(\forall x) \sim (\exists y)Fxy$	3 $\sim \exists$D
5.	Faa	2 $\exists$D
6.	$\sim (\exists y)Fay$ ✓	4 $\forall$D
7.	$(\forall y) \sim Fay$	6 $\sim \exists$D
8.	$\sim Faa$	7 $\forall$D
	$\times$	

The tree for the negation of '$(\exists x)Fxx \supset (\exists x)(\exists y)Fxy$' is closed. Therefore the latter sentence is quantificationally true.

c. 1.	$\sim ((\exists x)(\forall y)Lxy \supset (\exists x)Lxx)$ ✓	SM
2.	$(\exists x)(\forall y)Lxy$ ✓	1 $\sim \supset$D
3.	$\sim (\exists x)Lxx$ ✓	1 $\sim \supset$D
4.	$(\forall x) \sim Lxx$	3 $\sim \exists$D
5.	$(\forall y)Lay$	2 $\exists$D
6.	$\sim Laa$	4 $\forall$D
7.	Laa	5 $\forall$D
	$\times$	

The tree for the negation of '$(\exists x)(\forall y)Lxy \supset (\exists x)Lxx$' is closed. Therefore the latter sentence is quantificationally true.

e.	1.	$\sim ((\forall x)(Fx \supset (\exists y)Gya) \supset (Fb \supset (\exists y)Gya))$ ✓		SM
	2.	$(\forall x)(Fx \supset (\exists y)Gya)$		1 $\sim \supset$D
	3.	$\sim (Fb \supset (\exists y)Gya)$ ✓		1 $\sim \supset$D
	4.	Fb		3 $\sim \supset$D
	5.	$\sim (\exists y)Gya$ ✓		3 $\sim \supset$D
	6.	$(\forall y) \sim Gya$		5 $\sim \exists$D
	7.	$Fb \supset (\exists y)Gya$ ✓		2 $\forall$D
	8.	$\sim Fb$	$(\exists y)Gya$ ✓	7 $\supset$D
	9.	×	Gca	8 $\exists$D
	10.		$\sim Gca$	6 $\forall$D
			×	

The tree for the negation of '$(\forall x)(Fx \supset (\exists y)Gya) \supset (Fb \supset (\exists y)Gya)$' is closed. Therefore the latter sentence is quantificationally true.

g.	1.	$\sim ((\forall x)(Fx \supset (\forall y)Gxy) \supset (\exists x)(Fx \supset \sim (\forall y)Gxy))$ ✓		SM
	2.	$(\forall x)(Fx \supset (\forall y)Gxy)$		1 $\sim \supset$D
	3.	$\sim (\exists x)(Fx \supset \sim (\forall y)Gxy)$ ✓		1 $\sim \supset$D
	4.	$(\forall x) \sim (Fx \supset \sim (\forall y)Gxy)$		3 $\sim \exists$D
	5.	$\sim (Fa \supset \sim (\forall y)Gay)$ ✓		4 $\forall$D
	6.	Fa		5 $\sim \supset$D
	7.	$\sim \sim (\forall y)Gay$ ✓		5 $\sim \supset$D
	8.	$(\forall y)Gay$		7 $\sim \sim$ D
	9.	$Fa \supset (\forall y)Gay$ ✓		2 $\forall$D
	10.	$\sim Fa$	$(\forall y)Gay$	9 $\supset$D
	11.	×	Gaa	10 $\forall$D
			o	

1.	$(\forall x)(Fx \supset (\forall y)Gxy) \supset (\exists x)(Fx \supset \sim (\forall y)Gxy)$ ✓			SM
2.	$\sim (\forall x)(Fx \supset (\forall y)Gxy)$ ✓	$(\exists x)(Fx \supset \sim (\forall y)Gxy)$ ✓		1 $\supset$D
3.	$(\exists x) \sim (Fx \supset (\forall y)Gxy)$ ✓			2 $\sim \forall$D
4.	$\sim (Fa \supset (\forall y)Gay)$ ✓			3 $\exists$D
5.	Fa			4 $\sim \supset$D
6.	$\sim (\forall y)Gay$ ✓			4 $\sim \supset$D
7.	$(\exists y) \sim Gay$ ✓			6 $\sim \forall$D
8.	$\sim Gab$			7 $\exists$D
9.	o	$Fa \supset \sim (\forall y)Gay$ ✓		2 $\exists$D
10.		$\sim Fa$	$\sim (\forall y)Gay$ ✓	9 $\supset$D
11.		o	$(\exists y) \sim Gay$ ✓	10 $\sim \forall$D
12.			$\sim Gab$	11 $\exists$D
			o	

Both the tree for the given sentence and the tree for its negation have at least one completed open branch. Therefore the given sentence is quantificationally indeterminate.

4.a.

1.	~ ((∀x)Mxx ≡ ~ (∃x) ~ Mxx)✓		SM
2.	(∀x)Mxx	~ (∀x)Mxx✓	1 ~ ≡D
3.	~ ~ (∃x) ~ Mxx✓	~ (∃x) ~ Mxx✓	1 ~ ≡D
4.	(∃x) ~ Mxx✓		3 ~ ~ D
5.	~ Maa		4 ∃D
6.	Maa		2 ∀D
7.	×	(∃x) ~ Mxx✓	2 ~ ∀D
8.		(∀x) ~ ~ Mxx	3 ~ ∃D
9.		~ Mbb	7 ∃D
10.		~ ~ Mbb✓	8 ∀D
11.		Mbb	10 ~ ~ D
		×	

The tree is closed. Therefore the sentences '(∀x)Mxx' and '~ (∃x) ~ Mxx' are quantificationally equivalent.

c.

1.	~ ((∀x)(Fa ⊃ Gx) ≡ (Fa ⊃ (∀x)Gx))✓				SM
2.	(∀x)(Fa ⊃ Gx)		~ (∀x)(Fa ⊃ Gx)✓		1 ~ ≡D
3.	~ (Fa ⊃ (∀x)Gx)✓		Fa ⊃ (∀x)Gx✓		1 ~ ≡D
4.	Fa				3 ~ ⊃D
5.	~ (∀x)Gx✓				3 ~ ⊃D
6.	(∃x) ~ Gx✓				5 ~ ∀D
7.	~ Gb				6 ∃D
8.	Fa ⊃ Gb✓				2 ∀D
9.	~ Fa	Gb			8 ⊃D
10.	×	×	(∃x) ~ (Fa ⊃ Gx)✓		2 ~ ∀D
11.			~ (Fa ⊃ Gc)✓		10 ∃D
12.			Fa		11 ~ ⊃D
13.			~ Gc		11 ~ ⊃D
14.			~ Fa	(∀x)Gx	3 ⊃D
15.			×	Gc	14 ∀D
				×	

The tree is closed. Therefore the sentences '(∀x)(Fa ⊃ Gx)' and 'Fa ⊃ (∀x)Gx' are quantificationally equivalent.

e.	1.	$\sim (((\exists x)Fx \supset Ga) \equiv (\exists x)(Fx \supset Ga))\checkmark$				SM
	2.	$(\exists x)Fx \supset Ga\checkmark$		$\sim ((\exists x)Fx \supset Ga)\checkmark$		$1 \sim \equiv D$
	3.	$\sim (\exists x)(Fx \supset Ga)\checkmark$		$(\exists x)(Fx \supset Ga)\checkmark$		$1 \sim \equiv D$
	4.	$(\forall x) \sim (Fx \supset Ga)$				$3 \sim \exists D$
	5.	$\sim (\exists x)Fx\checkmark$	Ga			$2 \supset D$
	6.	$(\forall x) \sim Fx$				$5 \sim \exists D$
	7.	$\sim (Fa \supset Ga)\checkmark$	$\sim (Fa \supset Ga)\checkmark$			$4\ \forall D$
	8.	Fa	Fa			$7 \sim \supset D$
	9.	$\sim Ga$	$\sim Ga$			$7 \supset D$
	10.	$\sim Fa$	×			$6\ \forall D$
	11.	×		$(\exists x)Fx\checkmark$		$2 \sim \supset D$
	12.			$\sim Ga$		$2 \sim \supset D$
	13.			$Fb \supset Ga\checkmark$		$3\ \exists D$
	14.			Fc		$11\ \exists D$
	15.			$\sim Fb$	Ga	$13 \supset D$
				o	×	

The tree has a completed open branch. Therefore the given sentences are not quantificationally equivalent.

g.

1.	$\sim (((\forall x)Fx \supset Ga) \equiv (\exists x)(Fx \supset Ga))\checkmark$				SM
2.	$(\forall x)Fx \supset Ga\checkmark$		$\sim ((\forall x)Fx \supset Ga)\checkmark$		$1 \sim \equiv D$
3.	$\sim (\exists x)(Fx \supset Ga)\checkmark$		$(\exists x)(Fx \supset Ga)\checkmark$		$1 \sim \equiv D$
4.	$(\forall x) \sim (Fx \supset Ga)$				$3 \sim \exists D$
5.	$\sim (\forall x)Fx\checkmark$	Ga			$2 \supset D$
6.	$(\exists x) \sim Fx\checkmark$				$5 \sim \forall D$
7.	$\sim Fb$				$6\ \exists D$
8.	$\sim (Fb \supset Ga)\checkmark$	$\sim (Fa \supset Ga)\checkmark$			$4\ \forall D$
9.	Fb	Fa			$8 \sim \supset D$
10.	$\sim Ga$	$\sim Ga$			$8 \sim \supset D$
11.	$\times$	$\times$	$(\forall x)Fx$		$2 \sim \supset D$
12.			$\sim Ga$		$2 \sim \supset D$
13.			$Fc \supset Ga\checkmark$		$3\ \exists D$
14.			$\sim Fc$	Ga	$13 \supset D$
15.			Fc	$\times$	$11\ \forall D$
			$\times$		

The tree is closed. Therefore the sentences '$(\forall x)Fx \supset Ga$' and '$(\exists x)(Fx \supset Ga)$' are quantificationally equivalent.

i. 1.	$\sim ((\forall x)(\forall y)(Fx \supset Gy) \equiv (\forall x)(Fx \supset (\forall y)Gy))$ ✓			SM
2.	$(\forall x)(\forall y)(Fx \supset Gy)$		$\sim (\forall x)(\forall y)(Fx \supset Gy)$ ✓	$1 \sim \equiv D$
3.	$\sim (\forall x)(Fx \supset (\forall y)Gy)$ ✓		$(\forall x)(Fx \supset (\forall y)Gy)$	$1 \sim \equiv D$
4.	$(\exists x) \sim (Fx \supset (\forall y)Gy)$ ✓			$3 \sim \forall D$
5.	$\sim (Fa \supset (\forall y)Gy)$ ✓			$4\ \exists D$
6.	Fa			$5 \sim \supset D$
7.	$\sim (\forall y)Gy$ ✓			$5 \sim \supset D$
8.	$(\exists y) \sim Gy$ ✓			$7 \sim \forall D$
9.	$\sim Gb$			$8\ \exists D$
10.	$(\forall y)(Fa \supset Gy)$			$2\ \forall D$
11.	$Fa \supset Gb$ ✓			$10\ \forall D$
12.	$\sim Fa$	Gb		$11 \supset D$
13.	×	×	$(\exists x) \sim (\forall y)(Fx \supset Gy)$ ✓	$2 \sim \forall D$
14.			$\sim (\forall y)(Fc \supset Gy)$ ✓	$13\ \exists D$
15.			$(\exists y) \sim (Fc \supset Gy)$ ✓	$14 \sim \forall D$
16.			$\sim (Fc \supset Gd)$ ✓	$15\ \exists D$
17.			Fc	$16 \sim \supset D$
18.			$\sim Gd$	$16 \sim \supset D$
19.			$Fc \supset (\forall y)Gy$ ✓	$3\ \forall D$
20.			$\sim Fc$ $(\forall y)Gy$	$19 \supset D$
21.			× Gd	$20\ \forall D$
			×	

The tree is closed. Therefore the sentences '$(\forall x)(\forall y)(Fx \supset Gy)$' and '$(\forall x)(Fx \supset (\forall y)Gy)$' are quantificationally equivalent.

k. 1.					$\sim((\forall x)(Fa \equiv Gx) \equiv (Fa \equiv (\forall x)Gx))\checkmark$				SM
2.	$(\forall x)(Fa \equiv Gx)$				$\sim(\forall x)(Fa \equiv Gx)\checkmark$				$1 \sim\equiv D$
3.	$\sim(Fa \equiv (\forall x)Gx)\checkmark$				$Fa \equiv (\forall x)Gx\checkmark$				$1 \sim\equiv D$
4.	Fa		$\sim Fa$						$3 \sim\equiv D$
5.	$\sim(\forall x)Gx\checkmark$		$(\forall x)Gx$						$3 \sim\equiv D$
6.	$(\exists x)\sim Gx\checkmark$								$5 \sim\forall D$
7.	$\sim Gb$								$6\ \exists D$
8.	$Fa \equiv Gb\checkmark$		$Fa \equiv Gb\checkmark$						$2\ \forall D$
9.	Fa	$\sim Fa$	Fa	$\sim Fa$					$8 \equiv D$
10.	Gb	$\sim Gb$	Gb	$\sim Gb$					$8 \equiv D$
11.	$\times$	$\times$	$\times$	Gb					$5\ \forall D$
12.				$\times$	$(\exists x)\sim(Fa \equiv Gx)\checkmark$				$2 \sim\forall D$
13.					$\sim(Fa \equiv Gc)\checkmark$				$12\ \exists D$
									$5\ \forall D$
14.					Fa		$\sim Fa$		$3 \equiv D$
15.					$(\forall x)Gx$		$\sim(\forall x)Gx\checkmark$		$3 \equiv D$
16.					Fa	$\sim Fa$	Fa	$\sim Fa$	$13 \sim\equiv D$
17.					$\sim Gc$	Gc	$\sim Gc$	Gc	$13 \sim\equiv D$
18.					Gc	$\times$	$\times$		$15\ \forall D$
19.					$\times$			$(\exists x)\sim Gx\checkmark$	$15 \sim\forall D$
20.								$\sim Gd$	$19\ \exists D$

The tree has a completed open branch. Therefore the given sentences are not quantificationally equivalent.

m. 1.	$\sim ((\forall x)(Fx \supset (\forall y)Gy) \equiv (\forall x)(\forall y)(Fx \supset Gy))$ ✓				SM
2.	$(\forall x)(Fx \supset (\forall y)Gy)$		$\sim (\forall x)(Fx \supset (\forall y)Gy)$ ✓		$1 \sim \equiv D$
3.	$\sim (\forall x)(\forall y)(Fx \supset Gy)$ ✓		$(\forall x)(\forall y)(Fx \supset Gy)$		$1 \sim \equiv D$
4.	$(\exists x) \sim (\forall y)(Fx \supset Gy)$ ✓				$3 \sim \forall D$
5.	$\sim (\forall y)(Fa \supset Gy)$ ✓				$4\ \exists D$
6.	$(\exists y) \sim (Fa \supset Gy)$ ✓				$5 \sim \forall D$
7.	$\sim (Fa \supset Gb)$ ✓				$6\ \exists D$
8.	Fa				$7 \sim \supset D$
9.	$\sim Gb$				$7 \sim \supset D$
10.	$Fa \supset (\forall y)Gy$ ✓				$2\ \forall D$
11.	$\sim Fa$	$(\forall y)Gy$			$10 \supset D$
12.	×	Gb			$11\ \forall D$
13.		×	$(\exists x) \sim (Fx \supset (\forall y)Gy)$ ✓		$2 \sim \forall D$
14.			$\sim (Fc \supset (\forall y)Gy)$ ✓		$13\ \exists D$
15.			Fc		$14 \sim \supset D$
16.			$\sim (\forall y)Gy$ ✓		$14 \sim \supset D$
17.			$(\exists y) \sim Gy$ ✓		$16 \sim \forall D$
18.			$\sim Gd$		$17\ \exists D$
19.			$(\forall y)(Fc \supset Gy)$		$3\ \forall D$
20.			$Fc \supset Gd$ ✓		$19\ \forall D$
21.			$\sim Fc$	Gd	$20 \supset D$
			×	×	

The tree is closed. Therefore the sentences '$(\forall x)(Fx \supset (\forall y)Gy)$' and '$(\forall x)(\forall y)(Fx \supset Gy)$' are quantificationally equivalent.

5.a. 1.	$(\forall x)(Fx \supset Gx)$		SM
2.	Ga		SM
3.	$\sim Fa$		SM
4.	$Fa \supset Ga$ ✓		$1\ \forall D$
5.	$\sim Fa$	Ga	$4 \supset D$
	o	o	

The tree has at least one completed open branch. Therefore the argument is quantificationally invalid.

c.	1.	$(\forall x)(Kx \supset Lx)$	SM
	2.	$(\forall x)(Lx \supset Mx)$	SM
	3.	$\sim (\forall x)(Kx \supset Mx)$ ✓	SM
	4.	$(\exists x) \sim (Kx \supset Mx)$ ✓	$3 \sim \forall D$
	5.	$\sim (Ka \supset Ma)$ ✓	$4\ \exists D$
	6.	Ka	$5 \sim \supset D$
	7.	$\sim Ma$	$5 \sim \supset D$
	8.	$Ka \supset La$ ✓	$1\ \forall D$
	9.	$La \supset Ma$ ✓	$2\ \forall D$
	10.	$\sim Ka$ La	$8 \supset D$
		×	
	11.	$\sim La$ Ma	$9 \supset D$
		× ×	

The tree is closed. Therefore the argument is quantificationally valid.

e.	1.	$(\forall x)(Fx \supset Gx) \supset (\exists x)Nx$ ✓	SM
	2.	$(\forall x)(Nx \supset Gx)$	SM
	3.	$\sim (\forall x)(\sim Fx \vee Gx)$ ✓	SM
	4.	$(\exists x) \sim (\sim Fx \vee Gx)$ ✓	$3 \sim \forall D$
	5.	$\sim (\sim Fa \vee Ga)$ ✓	$4\ \exists D$
	6.	$\sim \sim Fa$ ✓	$5 \sim \vee D$
	7.	$\sim Ga$	$5 \sim \vee D$
	8.	Fa	$6 \sim \sim D$
	9.	$Na \supset Ga$ ✓	$2\ \forall D$
	10.	$\sim Na$ Ga	$9 \supset D$
		×	
	11.	$\sim (\forall x)(Fx \supset Gx)$ ✓ $(\exists x)Nx$ ✓	$1 \supset D$
	12.	Nb	$11\ \exists D$
	13.	$(\exists x) \sim (Fx \supset Gx)$ ✓	$11 \sim \forall D$
	14.	$\sim (Fb \supset Gb)$ ✓	$13\ \exists D$
	15.	Fb	$14 \sim \supset D$
	16.	$\sim Gb$	$14 \sim \supset D$
	17.	$Nb \supset Gb$ ✓ $Nb \supset Gb$ ✓	$2\ \forall D$
	18.	$\sim Nb$ Gb $\sim Nb$ Gb	$17 \supset D$
		o × × o	

The tree has at least one completed open branch. Therefore the argument is quantificationally invalid.

g. 1.	$(\forall x)(\sim Ax \supset Kx)$		SM
2.	$(\exists y) \sim Ky \checkmark$		SM
3.	$\sim (\exists w)(Aw \vee \sim Lwf) \checkmark$		SM
4.	$(\forall w) \sim (Aw \vee \sim Lwf)$		$3 \sim \exists D$
5.	$\sim Ka$		$2\ \exists D$
6.	$\sim Aa \supset Ka \checkmark$		$1\ \forall D$
7.	$\sim (Aa \vee \sim Laf) \checkmark$		$4\ \forall D$
8.	$\sim Aa$		$7 \sim \vee D$
9.	$\sim \sim Laf \checkmark$		$7 \sim \vee D$
10.	Laf		$9 \sim \sim D$
11.	$\sim \sim Aa \checkmark$	Ka	$6 \supset D$
12.	Aa	$\times$	$11 \sim \sim D$
	$\times$		

The tree is closed. Therefore the argument is quantificationally valid.

i. 1.	$(\forall x)(\forall y)Cxy$				SM
2.	$\sim ((Caa \ \&\ Cab) \ \&\ (Cba \ \&\ Cbb)) \checkmark$				SM
3.	$(\forall y)Cay$				$1\ \forall D$
4.	$(\forall y)Cby$				$1\ \forall D$
5.	Caa				$3\ \forall D$
6.	Cab				$3\ \forall D$
7.	Cba				$4\ \forall D$
8.	Cbb				$4\ \forall D$
9.	$\sim (Caa \ \&\ Cab) \checkmark$		$\sim (Cba \ \&\ Cbb) \checkmark$		$2 \sim \& D$
10.	$\sim Caa$	$\sim Cab$	$\sim Cba$	$\sim Cbb$	$9 \sim \& D$
	$\times$	$\times$	$\times$	$\times$	

The tree is closed. Therefore the argument is quantificationally valid.

k. 1.	$(\forall x)(Fx \supset Gx)$		SM
2.	$\sim (\exists x)Fx \checkmark$		SM
3.	$\sim \sim (\exists x)Gx \checkmark$		SM
4.	$(\exists x)Gx \checkmark$		$3 \sim \sim D$
5.	Ga		$4\ \exists D$
6.	$(\forall x) \sim Fx$		$2 \sim \exists D$
7.	$Fa \supset Ga \checkmark$		$1\ \forall D$
8.	$\sim Fa$		$6\ \forall D$
9.	$\sim Fa$	Ga	$7 \supset D$
	o	o	

The tree has at least one completed open branch. Therefore the argument is quantificationally invalid.

m.

1.	$(\exists x)Cx \supset Ch$ ✓		SM
2.	$\sim ((\exists x)Cx \equiv Ch)$ ✓		SM
3.	$(\exists x)Cx$ ✓	$\sim (\exists x)Cx$ ✓	$2 \sim \equiv D$
4.	$\sim Ch$	Ch	$2 \sim \equiv D$
5.		$(\forall x) \sim Cx$	$3 \sim \exists D$
6.		$\sim Ch$	$5\ \forall D$
7.	Ca	×	$3\ \exists D$

8.	$\sim (\exists x)Cx$ ✓	Ch	$1 \supset D$
9.	$(\forall x) \sim Cx$	×	$8 \sim \exists D$
10.	$\sim Ca$		$9\ \forall D$
	×		

The tree is closed. Therefore the argument is quantificationally valid.

6.a.

1.	$(\forall x) \sim Jx$		SM
2.	$(\exists y)(Hby \vee Ryy) \supset (\exists x)Jx$ ✓		SM
3.	$\sim (\forall y) \sim (Hby \vee Ryy)$ ✓		SM
4.	$(\exists y) \sim \sim (Hby \vee Ryy)$ ✓		$3 \sim \forall D$
5.	$\sim \sim (Hba \vee Raa)$ ✓		$4\ \exists D$
6.	$Hba \vee Raa$ ✓		$5 \sim \sim D$
7.	$\sim Ja$		$1\ \forall D$
8.	$\sim Jb$		$1\ \forall D$
9.	$\sim (\exists y)(Hby \vee Ryy)$ ✓	$(\exists x)Jx$ ✓	$2 \supset D$
10.		Jc	$9\ \exists D$
11.		$\sim Jc$	$1\ \forall D$
12.	$(\forall y) \sim (Hby \vee Ryy)$	×	$9 \sim \exists D$
13.	$\sim (Hba \vee Raa)$ ✓		$12\ \forall D$
14.	$\sim Hba$		$13 \sim \vee D$
15.	$\sim Raa$		$13 \sim \vee D$

16.	Hba	Raa	$6 \vee D$
	×	×	

The tree is closed. Therefore the entailment does hold.

c. 1.	$(\forall y)((Hy \,\&\, Fy) \supset Gy)$			SM
2.	$(\forall z)Fz \,\&\, \sim (\forall x)Kxb$ ✓			SM
3.	$\sim (\forall x)(Hx \supset Gx)$ ✓			SM
4.	$(\forall z)Fz$			2 &D
5.	$\sim (\forall x)Kxb$ ✓			2 &D
6.	$(\exists x) \sim (Hx \supset Gx)$ ✓			3 ~ ∀D
7.	$(\exists x) \sim Kxb$ ✓			5 ~ ∀D
8.	$\sim Kab$			7 ∃D
9.	$\sim (Hc \supset Gc)$ ✓			6 ∃D
10.	Hc			9 ~ ⊃D
11.	$\sim Gc$			9 ~ ⊃D
12.	$(Hc \,\&\, Fc) \supset Gc$ ✓			1 ∀D
13.	$\sim (Hc \,\&\, Fc)$ ✓		Gc	12 ⊃D
			×	
14.	$\sim Hc$	$\sim Fc$		13 ~ &D
15.	×	Fc		4 ∀D
		×		

The tree is closed. Therefore the entailment does hold.

e. 1.	$(\forall z)(Lz \equiv Hz)$		SM
2.	$(\forall x) \sim (Hx \vee \sim Bx)$		SM
3.	$\sim \sim Lb$ ✓		SM
4.	Lb		3 ~ ~ D
5.	$Lb \equiv Hb$ ✓		1 ∀D
6.	Lb	$\sim Lb$	5 ≡D
7.	Hb	$\sim Hb$	5 ≡D
8.	$\sim (Hb \vee \sim Bb)$ ✓	×	2 ∀D
9.	$\sim Hb$		8 ~ ∨D
10.	$\sim \sim Bb$		8 ~ ∨D
	×		

The tree is closed. Therefore the entailment does hold.

Section 9.4E

Note: Branches that are open but not completed are so indicated by a series of dots below the branch.

1.a.

1.	$(\forall x)Jx$				SM
2.	$(\forall x)(Jx \equiv (\exists y)(Gyx \vee Ky))$				SM
3.	Ja				1 $\forall$D
4.	$Ja \equiv (\exists y)(Gya \vee Ky)$ ✓				2 $\forall$D
5.	Ja		$\sim Ja$		4 $\equiv$D
6.	$(\exists y)(Gya \vee Ky)$ ✓		$\sim (\exists y)(Gya \vee Ky)$		4 $\equiv$D
			×		
7.	$Gaa \vee Ka$ ✓	$Gba \vee Kb$ ✓			6 $\exists$D2
8.	Gaa / Ka	Gba / Kb			7 $\vee$D
		⋮ / ⋮			

The tree has at least one completed open branch. Therefore the set is quantificationally consistent.

c.

1.	$(\exists x)Fx$ ✓		SM
2.	$(\exists x) \sim Fx$ ✓		SM
3.	Fa		1 $\exists$D2
4.	$\sim Fa$	$\sim Fb$	2 $\exists$D2
	×	o	

The tree has a completed open branch. Therefore the set is quantificationally consistent.

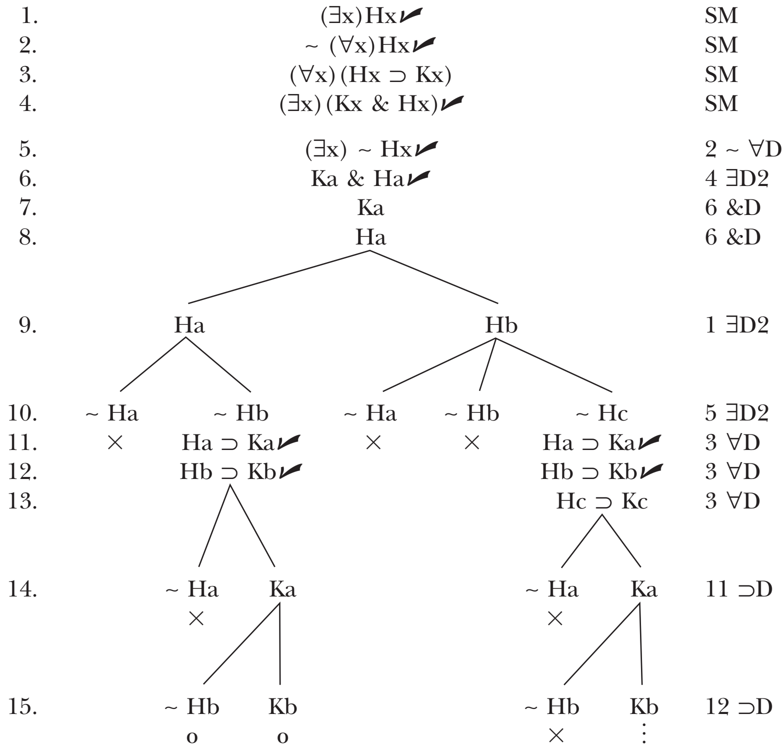
e. 1.		(∃x)Fx & (∃x) ~ Fx✓		SM
2.		(∃x)Fx ⊃ (∀x) ~ Fx✓		SM
3.		(∃x) Fx✓		1 &D
4.		(∃x) ~ Fx✓		1 &D
5.		Fa		3 ∃D2
6.	~ Fa	~ Fb		4 ∃D2
	×			
7.		~ (∃x)Fx✓	(∀x) ~ Fx	2 ⊃D
8.		(∀x) ~ Fx		7 ~ ∃D
9.		~ Fa		8 ∀D
		×	~ Fa	7 ∀D
			×	

The tree is closed. Therefore the set is quantificationally inconsistent.

g. 1.	(∀x)(∃y)Fxy		SM
2.	(∃y)(∀x) ~ Fyx✓		SM
3.	(∀x) ~ Fax		2 ∃D2
4.	(∃y)Fay✓		1 ∀D
5.	~ Faa		3 ∀D
6.	Faa	Fab	4 ∃D2
7.	×	(∃y)Fby	1 ∀D
8.		~ Fab	3 ∀D
		×	

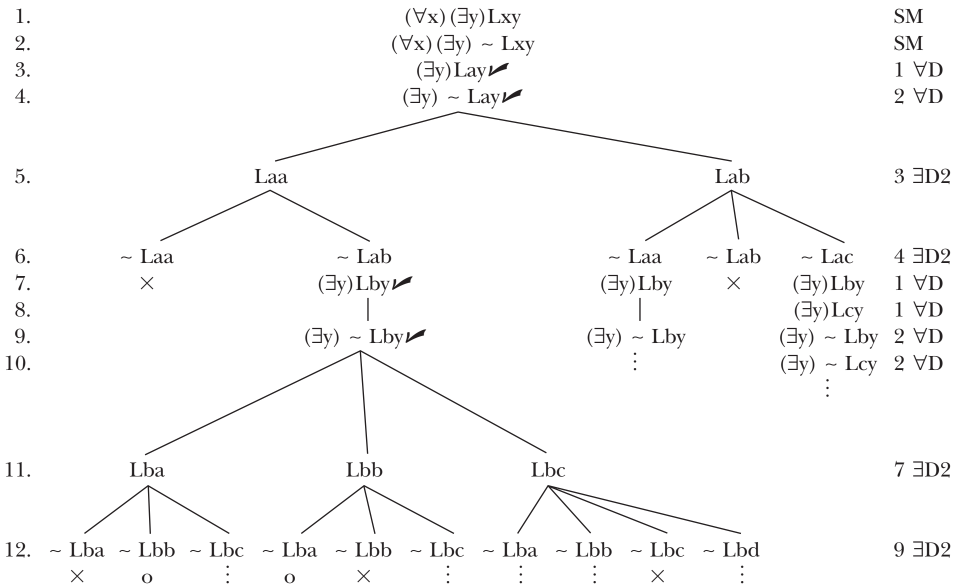
The tree is closed. Therefore the set is quantificationally inconsistent.

i.



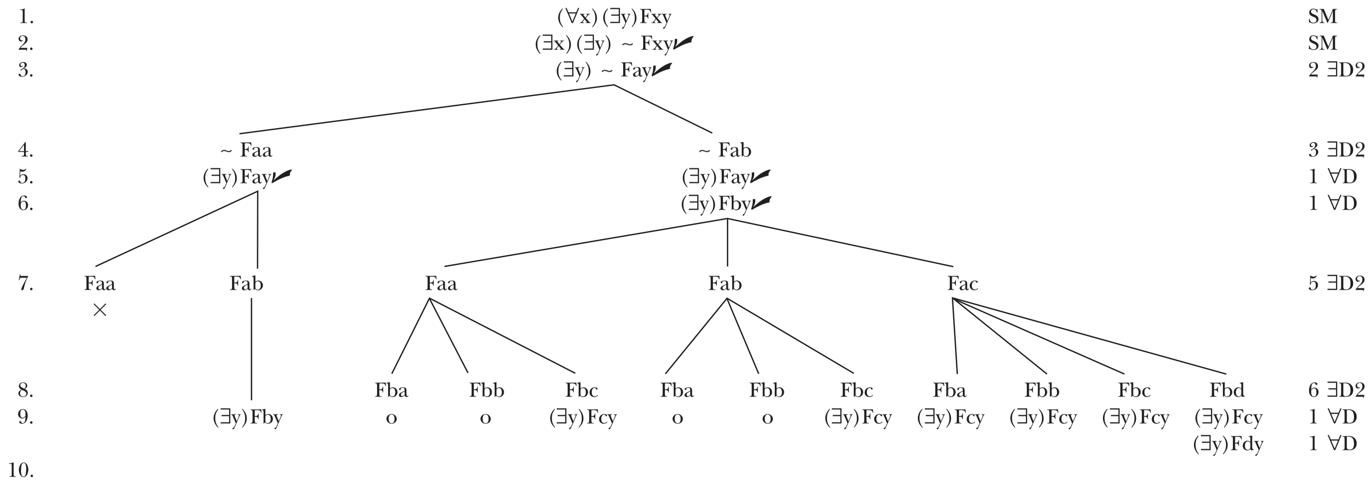
The tree has at least one completed open branch. The set is quantificationally consistent.

k.



The tree has at least one completed open branch. Therefore the set is quantificationally consistent.

m.



The tree has two completed open branch. Therefore the set is quantificationally consistent.

2.a.

1.	$(\forall x)(Fax \supset (\exists y)Fya)$		SM
2.	$Faa \supset (\exists y)Fya$ ✓		1 $\forall$D
3.	$\sim Faa$	$(\exists y)Fya$	2 $\supset$D
		⋮	

1.	$\sim (\forall x)(Fax \supset (\exists y)Fya)$ ✓		SM
2.	$(\exists x) \sim (Fax \supset (\exists y)Fya)$ ✓		1 $\sim \forall$D
3.	$\sim (Faa \supset (\exists y)Fya)$ ✓	$\sim (Fab \supset (\exists y)Fya)$ ✓	2 $\exists$D2
4.	Faa	Fab	3 $\sim \supset$D
5.	$\sim (\exists y)Fya$ ✓	$\sim (\exists y)Fya$ ✓	3 $\sim \supset$D
6.	$(\forall y) \sim Fya$	$(\forall y) \sim Fya$	5 $\sim \exists$D
7.	$\sim Faa$	$\sim Faa$	6 $\forall$D
8.	×	$\sim Fba$	6 $\forall$D
		o	

Both the tree for the sentence and the tree for its negation have at least one completed open branch. Therefore the sentence is quantificationally indeterminate.

c.

1.	$\sim (\forall x)(Fx \supset (\forall y)(Hy \supset Fy))$ ✓		SM
2.	$(\exists x) \sim (Fx \supset (\forall y)(Hy \supset Fy))$ ✓		1 $\sim \forall$D
3.	$\sim (Fa \supset (\forall y)(Hy \supset Fy))$ ✓		2 $\exists$D2
4.	Fa		3 $\sim \supset$D
5.	$\sim (\forall y)(Hy \supset Fy)$ ✓		3 $\sim \supset$D
6.	$(\exists y) \sim (Hy \supset Fy)$ ✓		5 $\sim \forall$D
7.	$\sim (Ha \supset Fa)$ ✓	$\sim (Hb \supset Fb)$ ✓	6 $\exists$D2
8.	Ha	Hb	7 $\sim \supset$D
9.	$\sim Fa$	$\sim Fb$	7 $\sim \supset$D
	×		

1.	$(\forall x)(Fx \supset (\forall y)(Hy \supset Fy))$		SM
2.	$Fa \supset (\forall y)(Hy \supset Fy)$ ✓		1 $\forall$D
3.	$\sim Fa$	$(\forall y)(Hy \supset Fy)$	2 $\supset$D
	o	⋮	

Both the tree for the sentence and the tree for its negation have at least one completed open branch. Therefore the sentence is quantificationally indeterminate.

e.

1.	~ ((∃x)(Fx ∨ ~ Fx) ≡ ((∃x)Fx ∨ (∃x) ~ Fx))✓				SM
2.	(∃x)(Fx ∨ ~ Fx)✓		~ (∃x)(Fx ∨ ~ Fx)✓		1 ~ ≡D
3.	~ ((∃x)Fx ∨ (∃x) ~ Fx)✓		(∃x)Fx ∨ (∃x) ~ Fx✓		1 ~ ≡D
4.	~ (∃x)Fx✓				3 ~ ∨D
5.	~ (∃x) ~ Fx✓				3 ~ ∨D
6.	(∀x) ~ Fx				4 ~ ∃D
7.	(∀x) ~ ~ Fx				5 ~ ∃D
8.	Fa ∨ ~ Fa✓				2 ∃D2
9.	Fa	~ Fa			
10.			(∀x) ~ (Fx ∨ ~ Fx)		2 ~ ∃D
11.			(∃x)Fx✓	(∃x) ~ Fx✓	3 ∨D
			Fa	~ Fa	11 ∃D2
12.	~ Fa	~ Fa			6 ∀D
13.	×	~ ~ Fa✓			7 ∀D
14.		Fa			13 ~ ~ D
15.		×	~ (Fa ∨ ~ Fa)✓	~ (Fa ∨ ~ Fa)✓	10 ∀D
16.			~ Fa	~ Fa	15 ~ ∨D
17.			~ ~ Fa	~ ~ Fa✓	15 ~ ∨D
18.			×	Fa	17 ~ ~ D
				×	

The tree for the negation of the sentence is closed. Therefore the sentence is quantificationally true.

g. 1.	$\sim ((\forall x)(Fx \supset ((\exists y)Gyx \supset H)) \supset (\forall x)(Fx \supset (\exists y)(Gyx \supset H)))$ ✓			SM
2.	$(\forall x)(Fx \supset ((\exists y)Gyx \supset H))$			$1 \sim \supset D$
3.	$\sim (\forall x)(Fx \supset ((\exists y)(Gyx \supset H))$ ✓			$1 \sim \supset D$
4.	$(\exists x) \sim (Fx \supset (\exists y)(Gyx \supset H))$ ✓			$3 \sim \forall D$
5.	$\sim (Fa \supset (\exists y)(Gya \supset H))$ ✓			$4\ \exists D2$
6.	Fa			$5 \sim \supset D$
7.	$\sim (\exists y)(Gya \supset H)$ ✓			$5 \sim \supset D$
8.	$(\forall y) \sim (Gya \supset H)$			$7 \sim \exists D$
9.	$\sim (Gaa \supset H)$ ✓			$8\ \forall D$
10.	$Fa \supset ((\exists y)Gya \supset H)$ ✓			$2\ \forall D$
11.	Gaa			$9 \sim \supset D$
12.	$\sim H$			$9 \sim \supset D$
13.	$\sim Fa$ ×	$(\exists y)Gya \supset H$ ✓		$10\ \supset D$
14.		$\sim (\exists y)Gya$ ✓	H ×	$13\ \supset D$
15.		$(\forall y) \sim Gya$		$14 \sim \exists D$
16.		$\sim Gaa$ ×		$15\ \forall D$

The tree for the negation of the sentence is closed. Therefore the sentence is quantificationally true.

3.a. 1.	Fa								SM
2.	$(\forall x)(Fx \supset Cx)$								SM
3.	$\sim (\forall x)(Fx \,\&\, Cx)$ ✓								SM
4.	$(\exists x) \sim (Fx \,\&\, Cx)$ ✓								$2 \sim \forall D$
5.	$\sim (Fa \,\&\, Ca)$ ✓			$\sim (Fb \,\&\, Cb)$ ✓					$4\ \exists D2$
6.	$\sim Fa$	$\sim Ca$		$\sim Fb$			$\sim Cb$		$5 \sim \&D$
7.	×	$Fa \supset Ca$ ✓		$Fa \supset Ca$ ✓			$Fa \supset Ca$ ✓		$2\ \forall D$
8.				$Fb \supset Cb$ ✓			$Fb \supset Cb$ ✓		$2\ \forall D$
9.		$\sim Fa$ ×	Ca ×	$\sim Fa$ ×	Ca		$\sim Fa$ ×	Ca	$7\ \supset D$
10.					$\sim Fb$ o	Cb o		$\sim Fb$ / Cb ×	$8\ \supset D$

The tree for the premises and the negation of the conclusion has at least one completed open branch. Therefore the argument is quantificationally invalid.

c. 1.	Fa			SM
2.	$(\forall x)(Fx \supset Cx)$			SM
3.	$\sim (\exists x)(Fx \,\&\, Cx)$ ✓			SM
4.	$(\forall x) \sim (Fx \,\&\, Cx)$			$3 \sim \exists D$
5.	$Fa \supset Ca$ ✓			$2\ \forall D$
6.	$\sim (Fa \,\&\, Ca)$ ✓			$4\ \forall D$
7.	$\sim Fa$	Ca		$5 \supset D$
	×			
8.		$\sim Fa$	$\sim Ca$	$6 \sim \&D$
		×	×	

The tree for the premises and the negation of the conclusion is closed. Therefore the argument is quantificationally valid.

e. 1.	$(\forall x)(\forall y)(\forall z)((Lxy \,\&\, Lyz) \supset Lxz)$				SM
2.	$(\forall x)(\forall y)(Lxy \supset Lyx)$				SM
3.	$\sim (\forall x)Lxx$ ✓				SM
4.	$(\exists x) \sim Lxx$ ✓				$3 \sim \forall D$
5.	$\sim Laa$				$4\ \exists D2$
6.	$(\forall y)(\forall z)((Lay \,\&\, Lyz) \supset Laz)$				$1\ \forall D$
7.	$(\forall y)(Lay \supset Lya)$				$2\ \forall D$
8.	$(\forall z)((Laa \,\&\, Laz) \supset Laz)$				$6\ \forall D$
9.	$Laa \supset Laa$ ✓				$7\ \forall D$
10.	$(Laa \,\&\, Laa) \supset Laa$ ✓				$8\ \forall D$
11.		$\sim (Laa \,\&\, Laa)$ ✓		Laa	$10 \supset D$
				×	
12.		$\sim Laa$	Laa		$9 \supset D$
			×		
13.	$\sim Laa$	$\sim Laa$			$11 \sim \&D$
	o	o			

The tree for the premises and the negation of the conclusion has at least one completed open branch. Therefore the argument is quantificationally invalid.

g. 1.		$(\exists x)((Lx \vee Sx) \vee Kx)$ ✓		SM
2.		$(\forall y) \sim (Ly \vee Ky)$		SM
3.		$\sim (\exists x)Sx$ ✓		SM
4.		$(La \vee Sa) \vee Ka$ ✓		1 ∃D2
5.		$(\forall x) \sim Sx$		3 ~ ∃D
6.	$La \vee Sa$ ✓		Ka	4 ∨D
7.	La	Sa		6 ∨D
8.	$\sim (La \vee Ka)$ ✓	$\sim (La \vee Ka)$	$\sim (La \vee Ka)$ ✓	2 ∀D
9.	$\sim Sa$	$\sim Sa$	$\sim Sa$	5 ∀D
10.	$\sim La$	×	$\sim La$	8 ~ ∨D
11.	$\sim Ka$		$\sim Ka$	8 ~ ∨D
	×		×	

The tree for the premises and the negation of the conclusion is closed. Therefore the argument is quantificationally valid.

i. 1.			$(\forall x)(Hx \supset Kcx)$			SM
2.			$(\forall x)(Lx \supset \sim Kcx)$			SM
3.			Ld			SM
4.			$\sim (\exists y) \sim Hy$ ✓			SM
5.			$(\forall y) \sim \sim Hy$			4 ~ ∃D
6.			$Hc \supset Kcc$ ✓			1 ∀D
7.			$Hd \supset Kcd$ ✓			1 ∀D
8.			$Lc \supset \sim Kcc$ ✓			2 ∀D
9.			$Ld \supset \sim Kcd$ ✓			2 ∀D
10.			$\sim \sim Hc$ ✓			5 ∀D
11.			$\sim \sim Hd$ ✓			5 ∀D
12.			Hc			10 ~ ~ D
13.			Hd			11 ~ ~ D
14.	$\sim Hc$ ×		Kcc			6 ⊃D
15.		$\sim Hd$ ×		Kcd		7 ⊃D
16.			$\sim Lc$		$\sim Kcc$ ×	8 ⊃D
17.			$\sim Ld$ ×	$\sim Kcd$ ×		9 ⊃D

The tree for the premises and the negation of the conclusion is closed. Therefore the argument is quantificationally valid.

6.a.	1.	~ ((∀x)(∀y) ~ Sxy ≡ ~ (∃x)(∃y)Sxy) ✓				SM
	2.	(∀x)(∀y) ~ Sxy		~ (∀x)(∀y) ~ Sxy ✓		1 ~ ≡D
	3.	~ ~ (∃x)(∃y)Sxy ✓		~ (∃x)(∃y)Sxy ✓		1 ~ ≡D
	4.	(∃x)(∃y)Sxy ✓				3 ~ ~ D
	5.	(∃y)Say ✓				4 ∃D2
	6.	Saa	Sab			5 ∃D2
	7.			(∃x) ~ (∀y) ~ Sxy ✓		2 ~ ∀D
	8.			(∀x) ~ (∃y)Sxy		3 ~ ∃D
	9.			~ (∀y) ~ Say ✓		7 ∃D2
	10.			(∃y) ~ ~ Say ✓		9 ~ ∀D
	11.			~ ~ Saa ✓	~ ~ Sab ✓	10 ∃D2
	12.			Saa	Sab	11 ~ ~ D
	13.			~ (∃y)Say ✓	~ (∃y)Say ✓	8 ∀D
	14.				~ (∃y)Sby ✓	8 ∀D
	15.	(∀y) ~ Say	(∀y) ~ Say			2 ∀D
	16.		(∀y) ~ Sby			2 ∀D
	17.	~ Saa	~ Sab			15 ∀D
	18.	×	×	(∀y) ~ Say	(∀y) ~ Say	13 ~ ∃D
	19.				(∀y) ~ Sby	14 ~ ∃D
	20.			~ Saa	~ Sab	18 ∀D
				×	×	

The tree for the negation of the corresponding biconditional is closed. Therefore the sentences are equivalent.

c.

1.	~ ((∃x)(∀x ⊃ B) ≡ ((∀x)Ax ⊃ B))✓				SM
2.	(∃x)(Ax ⊃ B)✓		~ (∃x)(Ax ⊃ B)✓		1 ~ ≡D
3.	~ ((∀x)Ax ⊃ B)✓		(∀x)Ax ⊃ B✓		1 ~ ≡D
4.	(∀x)Ax				3 ~ ⊃D
5.	~ B				3 ~ ⊃D
6.	Aa ⊃ B✓				2 ∃D2
7.	~ Aa	B			6 ⊃D
8.		×	(∀x) ~ (Ax ⊃ B)		2 ~ ∃D
9.			~ (∀x)Ax✓	B	3 ⊃D
10.			(∃x) ~ Ax✓		9 ~ ∀D
11.			~ Aa		10 ∃D2
12.	Aa				4 ∀D
13.	×		~ (Aa ⊃ B)✓	~ (Aa ⊃ B)✓	8 ∀D
14.			Aa	Aa	13 ~ ⊃D
15.			~ B	~ B	13 ~ ⊃D
			×	×	

The tree for the negation of the corresponding biconditional is closed. Therefore the sentences are equivalent.

e.

1.	~ ((∀x)(Ax ⊃ B) ≡ ((∃x)Ax ⊃ B))✓				SM
2.	(∀x)(Ax ⊃ B)		~ (∀x)(Ax ⊃ B)✓		1 ~ ≡D
3.	~ ((∃x)Ax ⊃ B)✓		(∃x)Ax ⊃ B✓		1 ~ ≡D
4.	(∃x)Ax✓				3 ~ ⊃D
5.	~ B				3 ~ ⊃D
6.	Aa				4 ∃D2
7.			(∃x) ~ (Ax ⊃ B)✓		2 ~ ∀D
8.			~ (Aa ⊃ B)✓		7 ∃D2
9.			Aa		8 ~ ⊃D
10.			~ B		8 ~ ⊃D
11.			~ (∃x)Ax✓	B	3 ⊃D
12.			(∀x) ~ Ax	×	11 ~ ∃D
13.			~ Aa		12 ∀D
14.	Aa ⊃ B✓		×		2 ∀D
15.	~ Aa	B			14 ⊃D
	×	×			

The tree for the negation of the corresponding biconditional is closed. Therefore the sentences are equivalent.

g.					
1.	$\sim((\exists x)(\exists y)Hxy \equiv (\exists y)(\exists x)Hxy)\checkmark$				SM
2.	$(\exists x)(\exists y)Hxy\checkmark$		$\sim(\exists x)(\exists y)Hxy\checkmark$		$1 \sim\equiv D$
3.	$\sim(\exists y)(\exists x)Hxy\checkmark$		$(\exists y)(\exists x)Hxy\checkmark$		$1 \sim\equiv D$
4.	$(\exists y)Hay\checkmark$				$2\ \exists D2$
5.	$(\forall y)\sim(\exists x)Hxy$				$3 \sim\exists D$
6.			$(\exists x)Hxa\checkmark$		$3\ \exists D2$
7.	Haa	Hab			$4\ \exists D2$
8.			$(\forall x)\sim(\exists y)Hxy$		$2 \sim\exists D$
9.			Haa	Hba	$6\ \exists D2$
10.	$\sim(\exists x)Hxa\checkmark$	$\sim(\exists x)Hxa\checkmark$			$5\ \forall D$
11.		$\sim(\exists x)Hxb\checkmark$			$5\ \forall D$
12.			$\sim(\exists y)Hay\checkmark$	$\sim(\exists y)Hay\checkmark$	$8\ \forall D$
13.				$\sim(\exists y)Hby\checkmark$	$8\ \forall D$
14.	$(\forall x)\sim Hxa$	$(\forall x)\sim Hxa$			$10 \sim\exists D$
15.		$(\forall x)\sim Hxb$			$11 \sim\exists D$
16.			$(\forall y)\sim Hay$	$(\forall y)\sim Hay$	$12 \sim\exists D$
17.				$(\forall y)\sim Hby$	$13 \sim\exists D$
18.	$\sim Haa$	$\sim Haa$			$14\ \forall D$
19.	×	$\sim Hab$			$15\ \forall D$
20.		×	$\sim Haa$		$16\ \forall D$
21.			×	$\sim Hba$	$17\ \forall D$
				×	

The tree for the negation of the corresponding biconditional is closed. Therefore the sentences are equivalent.

5.a.				
1.	$(\forall x)(Fax \supset Fxa)$			SM
2.	$\sim(Fab \vee Fba)\checkmark$			SM
3.	$\sim Fab$			$2 \sim\vee D$
4.	$\sim Fba$			$2 \sim\vee D$
5.	$Faa \supset Faa\checkmark$			$1\ \forall D$
6.	$Fab \supset Fba\checkmark$			$1\ \forall D$
7.	$\sim Fab$		Fba	$6 \supset D$
			×	
8.	$\sim Faa$	Faa		$5 \supset D$
	o	o		

The tree has at least one completed open branch. Therefore the given set does not quantificationally entail the given sentence.

c. 1.	$\sim Fa$		SM
2.	$(\forall x)(Fa \supset (\exists y)Gxy)$		SM
3.	$\sim\sim(\exists y)Gay \checkmark$		SM
4.	$(\exists y)Gay \checkmark$		$3 \sim\sim D$
5.	Gaa	Gab	$4\ \exists D2$
6.	$Fa \supset (\exists y)Gay \checkmark$	$Fa \supset (\exists y)Gay \checkmark$	$2\ \forall D$
7.		$Fb \supset (\exists y)Gby \checkmark$	$2\ \forall D$
		$\vdots$	

8.	$\sim Fa$	$(\exists y)Gay$	$6 \supset D$
	o	$\vdots$	

The tree has at least one completed open branch. Therefore the given set does not quantificationally entail the given sentence.

e. 1.	$(\exists x)Gx \checkmark$				SM
2.	$(\forall x)(Gx \supset Dxx)$				SM
3.	$\sim(\exists x)(Gx \,\&\, (\forall y)Dxy) \checkmark$				SM
4.	$(\forall x)\sim(Gx \,\&\, (\forall y)Dxy) \checkmark$				$3 \sim\exists D$
5.	Ga				$1\ \exists D2$
6.	$Ga \supset Daa \checkmark$				$2\ \forall D$
7.	$\sim(Ga \,\&\, (\forall y)Day) \checkmark$				$4\ \forall D$
8.	$\sim Ga$		$\sim(\forall y)Day \checkmark$		$7 \sim \& D$
	$\times$				
9.		$\sim Ga$		Daa	$6 \supset D$
10.		$\times$		$(\exists y)\sim Day \checkmark$	$8 \sim\forall D$
11.	$\sim Daa$			$\sim Dab$	$10\ \exists D2$
12.	$\times$			$Gb \supset Dbb \checkmark$	$2\ \forall D$
13.				$\sim(Gb \,\&\, (\forall y)Dby) \checkmark$	$4\ \forall D$
14.		$\sim Gb$		$\sim(\forall y)Dby$	$13 \sim \& D$
15.	$\sim Gb$	Dbb	$\sim Gb$	Dbb	$12 \supset D$
	o	o	$\vdots$	$\vdots$	

The tree has at least one completed open branch. Therefore the given set does not quantificationally entail the given sentence.

7. If a tree is closed, then on each branch of that tree there is some atomic sentence **P** and its negation, ~ **P**. One of these sentences occurs subsequent to the other on the branch in question. Let **Q** be the latter of the two sentences and let **n** be the number of the line on which **Q** occurs. Then **n** is either the last line of the branch or the second to the last line of the branch. The reason is that once both an atomic sentence and its negation have been added to a branch, that branch is closed and no further sentences can be added to the branch after the current decomposition has been completed. (Some decomposition rules do add two sentences to each branch passing through the sentence being decomposed.) Hence such a branch is finite—for no infinite branch can have a last member.

9. No. For example, consider the sentence '(∃x)(Fx & ~ Fb)' and its substitution instance 'Fb & ~ Fb'. Clearly, every tree for the unit set of the latter sentence closes, but the systematic tree for the unit set of '(∃x)(Fx & ~ Fb)' does not close. Rather it has a completed open branch:

1.	(∃x)(Fx & ~ Fb)✓		SM
2.	Fa & ~ Fb✓	Fb & ~ Fb✓	1 ∃D2
3.	Fa	Fb	2 &D
4.	~ Fb	~ Fb	2 &D
		×	

11. Since it is already specified that stage 1 is done before stage 2 and stage 2 before stage 3, and stage 3 before stage 4, we would have to specify the order in which work within each stage is to be done, and what constants are to be used in what order.

Section 9.5E

1.a.

1.	(∀x)Fxx	SM
2.	(∃x)(∃y) ~ Fxy✓	SM
3.	(∀x)x = a	SM
4.	(∃y) ~ Fby✓	2 ∃D
5.	~ Fbc	4 ∃D
6.	Faa	1 ∀D
7.	c = a	3 ∀D
8.	Fac	6, 7 =D
9.	b = a	3 ∀D
10.	Fbc	8, 9 =D
	×	

The tree is closed. Therefore the set is quantificationally inconsistent.

c. 1.	$(\forall x)(x = a \supset Gxb)$		SM
2.	$\sim (\exists x)Gxx$ ✓		SM
3.	$a = b$		SM
4.	$(\forall x) \sim Gxx$		$2 \sim \exists D$
5.	$a = a \supset Gab$ ✓		$1\ \forall D$
6.	$\sim a = a$	Gab	$5 \supset D$
7.	×	$\sim Gaa$	$4\ \forall D$
8.		Gaa	$3, 6\ {=}D$
		×	

The tree is closed. Therefore the set is quantificationally inconsistent.

e. 1.	$(\forall x)((Fx \,\&\, \sim Gx) \supset \sim x = a)$			SM
2.	$Fa \,\&\, \sim Ga$ ✓			SM
3.	Fa			$2\ \&D$
4.	$\sim Ga$			$2\ \&D$
5.	$(Fa \,\&\, \sim Ga) \supset \sim a = a$ ✓			$1\ \forall D$
6.	$\sim (Fa \,\&\, \sim Ga)$ ✓		$\sim a = a$	$5 \supset D$
			×	
7.	$\sim Fa$	$\sim \sim Ga$ ✓		$6 \sim \&D$
8.	×	Ga		$7 \sim \sim D$
		×		

The tree is closed. Therefore the set is quantificationally inconsistent.

g. 1.	$(\forall x)(x = a \supset Gxf(b))$		SM
2.	$\sim (\exists x)Gxf(x)$ ✓		SM
3.	$f(a) = f(b)$		SM
4.	$(\forall x) \sim Gxf(x)$		$2 \sim \exists D$
5.	$a = a \supset Gaf(b)$ ✓		$1\ \forall D$
6.	$\sim a = a$	$Gaf(b)$	$5 \supset D$
7.	×	$\sim Gaf(a)$	$4\ \forall D$
8.		$Gaf(a)$	$3, 6\ {=}D$
		×	

The tree is closed. Therefore the set is quantificationally inconsistent.

i.

1.	$(\exists x) \sim x = g(x)$ ✓	SM
2.	$(\forall x)(\forall y)x = g(y)$	SM
3.	$\sim a = g(a)$	1 $\exists$D
4.	$(\forall y)a = g(y)$	2 $\forall$D
5.	$a = g(a)$	4 $\forall$D
	$\times$	

The tree is closed. Therefore the set is quantificationally inconsistent.

k.

1.	$(\forall x)[Hx \supset (\forall y)Txy]$		SM
2.	$(\exists x)Hf(x)$ ✓		SM
3.	$\sim (\exists x)Txx$ ✓		SM
4.	$Hf(a)$		2 $\exists$D
5.	$(\forall x) \sim Txx$		3 $\sim \exists$D
6.	$Hf(a) \supset (\forall y)Tf(a)y$ ✓		1 $\forall$D
			6 $\supset$D
7.	$\sim Hf(a)$	$(\forall y)Tf(a)y)$	
	$\times$	$Tf(a)f(a)$	7 $\forall$D
		$\sim Tf(a)f(a)$	5 $\forall$D
		$\times$	

The tree is closed. Therefore the set is quantificationally inconsistent.

m.

1.	$(\exists x)Fx \supset (\exists x)(\exists y)f(y) = x$ ✓		SM
2.	$(\exists x)Fx$ ✓		SM
3.	Fa		2 $\exists$D
4.	$\sim(\exists x)Fx$ ✓	$(\exists x)(\exists y)f(y) = x$ ✓	1 $\supset$D
5.	$(\forall x)\sim Fx$		4 $\sim \exists$
6.	$\sim Fa$		5 $\forall$D
7.	$\times$	$(\exists y)f(y) = b$ ✓	4 $\exists$D
8.		$f(c) = b$	7 $\exists$D
		o	

The tree has a completed open branch. Therefore the set is quantificationally consistent.

The literals 'Fa', and '$f(c) = b$' on the completed open branch will be be true on any interpretation that makes the following assignments:

UD:	{2, 4, 6}
a:	6
b:	4
c:	2
f(x):	x^2
Fx:	x is even

2. a.

1.	$\sim (a = b \equiv b = a)$ ✓		SM
2.	$a = b$	$\sim a = b$	$1 \sim \equiv D$
3.	$\sim b = a$	$b = a$	$1 \sim \equiv D$
4.	$\sim a = a$	$\sim b = b$	$2, 3 = D$
	×	×	

The tree is closed. Therefore ‘a = b ≡ b = a’ is quantificationally true.

c.

1.	$\sim ((Gab \;\&\; \sim Gba) \supset \sim a = b)$ ✓	SM
2.	$Gab \;\&\; \sim Gba$ ✓	$1 \sim \supset D$
3.	$\sim \sim a = b$ ✓	$1 \sim \supset D$
4.	Gab	2 &D
5.	$\sim Gba$	2 &D
6.	$a = b$	$3 \sim \sim D$
7.	Gaa	$4, 6 = D$
8.	$\sim Gaa$	$5, 6 = D$
	×	

The tree is closed. Therefore the sentence ‘(Gab & ~ Gba) ⊃ ~ a = b’ is quantificationally true.

e.

1.	$\sim (Fa \equiv (\exists x)(Fx \;\&\; x = a))$ ✓			SM
2.	Fa		$\sim Fa$	$1 \sim \equiv D$
3.	$\sim (\exists x)(Fx \;\&\; x = a)$ ✓		$(\exists x)(Fx \;\&\; x = a)$ ✓	$1 \sim \equiv D$
4.	$(\forall x) \sim (Fx \;\&\; x = a)$			$3 \sim \exists D$
5.	$\sim (Fa \;\&\; a = a)$ ✓			$4\ \forall D$
6.	$\sim Fa$	$\sim a = a$		$5 \sim \&D$
7.	×	×	$Fb \;\&\; b = a$ ✓	$3\ \exists D$
8.			Fb	7 &D
9.			$b = a$	7 &D
10.			$\sim Fb$	$2, 9 = D$
			×	

The tree is closed. Therefore the sentence ‘Fa ≡ (∃x)(Fx & x = a)’ is quantificationally true.

g.	1.	$\sim((\forall x)x = a \supset ((\exists x)Fx \supset (\forall x)Fx))$ ✓	SM
	2.	$(\forall x)x = a$	$1 \sim\supset$D
	3.	$\sim((\exists x)Fx \supset (\forall x)Fx)$ ✓	$1 \sim\supset$D
	4.	$(\exists x)Fx$ ✓	$3 \sim\supset$D
	5.	$\sim(\forall x)Fx$ ✓	$3 \sim\supset$D
	6.	$(\exists x) \sim Fx$ ✓	$5 \sim\forall$D
	7.	Fb	4 $\exists$D
	8.	$\sim Fc$	6 $\exists$D
	9.	$c = a$	2 $\forall$D
	10.	$b = a$	2 $\forall$D
	11.	$c = b$	9, 10 =D
	12.	Fc	7, 11 =D
		×	

The tree is closed. Therefore the sentence '$(\forall x)x = a \supset ((\exists x)Fx \supset (\forall x)Fx)$' is quantificationally true.

i.	1.	$(\forall x)(\forall y) \sim x = y$	SM
	2.	$(\forall y) \sim a = y$	1 $\forall$D
	3.	$\sim a = a$	2 $\forall$D
		×	

The tree is closed. Therefore the sentence '$(\forall x)(\forall y) \sim x = y$' is quantificationally false.

k.	1.	$(\exists x)(\exists y) \sim x = y$ ✓	SM
	2.	$(\exists y) \sim a = y$ ✓	1 $\exists$D
	3.	$\sim a = b$	2 $\exists$D

1.	$\sim(\exists x)(\exists y) \sim x = y$ ✓	SM
2.	$(\forall x) \sim (\exists y) \sim x = y$	$1 \sim\exists$D
3.	$\sim(\exists y) \sim a = y$ ✓	2 $\forall$D
4.	$(\forall y) \sim\sim a = y$	$3 \sim\exists$D
5.	$\sim\sim a = a$ ✓	4 $\forall$D
6.	$a = a$	$5 \sim\sim$ D
	o	

Both the tree for the given sentence and the tree for its negation have at least one completed open branch. Therefore the given sentence is quantificationally indeterminate.

m. 1.	$\sim (\forall x)(\forall y)((Fx \equiv Fy) \supset x = y)$ ✓		SM
2.	$(\exists x) \sim (\forall y)((Fx \equiv Fy) \supset x = y)$ ✓		$1 \sim \forall D$
3.	$\sim (\forall y)((Fa \equiv Fy) \supset a = y)$ ✓		$2\ \exists D$
4.	$(\exists y) \sim ((Fa \equiv Fy) \supset a = y)$ ✓		$3 \sim \forall D$
5.	$\sim ((Fa \equiv Fb) \supset a = b)$ ✓		$4\ \exists D$
6.	$Fa \equiv Fb$ ✓		$5 \sim \supset D$
7.	$\sim a = b$		$5 \sim \supset D$
8.	Fa	$\sim Fa$	$6 \equiv D$
9.	Fb	$\sim Fb$	$6 \equiv D$

1.	$(\forall x)(\forall y)((Fx \equiv Fy) \supset x = y)$			SM
2.	$(\forall y)((Fa \equiv Fy) \supset a = y)$			$1\ \forall D$
3.	$(Fa \equiv Fa) \supset a = a$ ✓			$2\ \forall D$
4.	$\sim (Fa \equiv Fa)$ ✓		$a = a$	$3 \supset D$
			o	
5.	Fa	$\sim Fa$		$4 \sim \equiv D$
6.	$\sim Fa$	Fa		$4 \sim \equiv D$
	×	×		

Both the tree for the given sentence and the tree for its negation have at least one completed open branch. Therefore the given sentence is quantificationally indeterminate.

o. 1.	$\sim (((\exists x)Gax \,\&\, \sim (\exists x)Gxa) \supset (\forall x)(Gxa \supset \sim x = a))$ ✓	SM
2.	$(\exists x)Gax \,\&\, \sim (\exists x)Gxa$ ✓	$1 \sim \supset D$
3.	$\sim (\forall x)(Gxa \supset \sim x = a)$ ✓	$1 \sim \supset D$
4.	$(\exists x)Gax$ ✓	$2\ \&D$
5.	$\sim (\exists x)Gxa$ ✓	$2\ \&D$
6.	$(\forall x) \sim Gxa$	$5 \sim \exists D$
7.	$(\exists x) \sim (Gxa \supset \sim x = a)$ ✓	$3 \sim \forall D$
8.	$\sim (Gba \supset \sim b = a)$ ✓	$7\ \exists D$
9.	Gac	$4\ \exists D$
10.	Gba	$8 \sim \supset D$
11.	$\sim \sim b = a$	$8 \sim \supset D$
12.	$\sim Gba$	$6\ \forall D$
	×	

The tree is closed. Therefore the sentence '$[(\exists x)Gax \,\&\, \sim (\exists x)Gxa] \supset (\forall x)(Gxa \supset \sim x = a)$' is quantificationally true.

3. a.	1.	$\sim (\exists x)x = f(a)$	SM
	2.	$(\forall x) \sim x = f(a)$	$1 \sim \exists D$
	3.	$\sim f(a) = f(a)$	$2\ \forall D$
		$\times$	

The tree is closed. Therefore the given sentence is quantificationally true.

c.	1.	$\sim (\exists x)(\exists y)x = y$	SM
	2.	$(\forall x) \sim (\exists y)x = y$	$1 \sim \exists D$
	3.	$\sim (\exists y)a = y$	$2\ \forall D$
	4.	$(\forall y) \sim a = y$	$3 \sim \exists D$
	5.	$\sim a = a$	$4\ \forall D$
		$\times$	

The tree is closed. Therefore the given sentence is quantificationally true.

e.	1.	$\sim (\forall x)[Gx \supset (\exists y)f(x) = y]$	SM
	2.	$(\exists x) \sim [Gx \supset (\exists y)f(x) = y]$	$1 \sim \forall D$
	3.	$\sim [Ga \supset (\exists y)f(a) = y]$	$2\ \exists D$
	4.	Ga	$3 \sim \supset D$
	5.	$\sim (\exists y)f(a) = y$	$3 \sim \supset D$
	6.	$(\forall y) \sim f(a) = y$	$5 \sim \exists D$
	7.	$\sim f(a) = f(a)$	$7\ \forall D$
		$\times$	

The tree is closed. Therefore the given sentence is quantificationally true.

g.	1.	$\sim (\forall y) \sim [(\forall x)x = y \vee (\forall x)f(x) = y]$ ✓		SM
	2.	$(\exists y) \sim \sim [(\forall x)x = y \vee (\forall x)f(x) = y]$ ✓		$1 \sim \forall D$
	3.	$\sim \sim [(\forall x)x = a \vee (\forall x)f(x) = a]$ ✓		$2\ \exists D$
	4.	$[(\forall x)x = a \vee (\forall x)f(x) = a]$ ✓		$3 \sim \sim \exists D$
	5.	$(\forall x)x = a$	$(\forall x)f(x) = a$	$4 \vee D$
	6.	$a = a$	$f(a) = a$	$5\ \forall D$
		o		

The tree has a completed open branch. Therefore the given sentence is not quantificationally true.

4.a.

1.	~ (~ a = b ≡ ~ b = a) ✓		SM
2.	~ a = b	~ ~ a = b ✓	1 ~ ≡D
3.	~ ~ b = a ✓	~ b = a	1 ~ ≡D
4.	b = a		3 ~ ~ D
5.	~ b = b		2, 4 = D
6.	×	a = b	2 ~ ~ D
7.		~ a = a	6, 3 =D
		×	

The tree is closed. Therefore the sentences '~ a = b' and '~ b = a' are quantificationally equivalent.

c.

1.	~ ((∀x)x = a ≡ (∀x)x = b) ✓		SM
2.	(∀x)x = a	~ (∀x)x = a ✓	1 ~ ≡D
3.	~ (∀x)x = b ✓	(∀x)x = b	1 ~ ≡D
4.	(∃x) ~ x = b ✓		3 ~ ∀D
5.	~ c = b		4 ∃D
6.	b = a		2 ∀D
7.	c = a		2 ∀D
8.	c = b		6, 7 =D
9.	×	(∃x) ~ x = a ✓	2 ~ ∀D
10.		~ c = a	9 ∃D
11.		c = b	3 ∀D
12.		a = b	3 ∀D
13.		c = a	11, 12 =D
		×	

The tree is closed. Therefore the sentences '(∀x)x = a' and '(∀x)x = b' are quantificationally equivalent.

e.	1.	$\sim ((\forall x)(\forall y)x = y \equiv (\forall x)x = a)$ ✓		SM
	2.	$(\forall x)(\forall y)x = y$	$\sim (\forall x)(\forall y)x = y$ ✓	$1 \sim \equiv D$
	3.	$\sim (\forall x)x = a$ ✓	$(\forall x)x = a$	$1 \sim \equiv D$
	4.	$(\exists x) \sim x = a$ ✓		$3 \sim \forall D$
	5.	$\sim b = a$		$4\ \exists D$
	6.	$(\forall y)b = y$		$2\ \forall D$
	7.	$b = a$		$6\ \forall D$
	8.	×	$(\exists x) \sim (\forall y)x = y$ ✓	$2 \sim \forall D$
	9.		$\sim (\forall y)b = y$ ✓	$8\ \exists D$
	10.		$(\exists y) \sim b = y$ ✓	$9 \sim \forall D$
	11.		$\sim b = c$	$10\ \exists D$
	12.		$b = a$	$3\ \forall D$
	13.		$c = a$	$3\ \forall D$
	14.		$b = c$	$12, 13 = D$
			×	

The tree is closed. Therefore the sentences '$(\forall x)(\forall y)x = y$' and '$(\forall x)x = a$' are quantificationally equivalent.

g.	1.	$\sim ((\forall x)(Fx \supset x = a) \equiv (\forall x)(Fa \supset x = a))$ ✓		SM
	2.	$(\forall x)(Fx \supset x = a)$	$\sim (\forall x)(Fx \supset x = a)$ ✓	$1 \sim \equiv D$
	3.	$\sim (\forall x)(Fa \supset x = a)$ ✓	$(\forall x)(Fa \supset x = a)$	$1 \sim \equiv D$
	4.	$(\exists x) \sim (Fa \supset x = a)$ ✓		$3 \sim \forall D$
	5.	$\sim (Fa \supset b = a)$ ✓		$4\ \exists D$
	6.	Fa		$5 \sim \supset D$
	7.	$\sim b = a$		$5 \sim \supset D$
	8.	$Fb \supset b = a$ ✓		$2\ \forall D$
	9.	$\sim Fb$ $b = a$		$8 \supset D$
	10.	$Fa \supset a = a$ ✓ ×		$2\ \forall D$
	11.	$\sim Fa$ $a = a$		$10 \supset D$
	12.	× o	$(\exists x) \sim (Fx \supset x = a)$ ✓	$2 \sim \forall D$
	13.		$\sim (Fb \supset b = a)$ ✓	$12\ \exists D$
	14.		Fb	$13 \sim \supset D$
	15.		$\sim b = a$	$13 \sim \supset D$
	16.		$Fa \supset a = a$ ✓	$3\ \forall D$
	17.		$Fa \supset b = a$ ✓	$3\ \forall D$
	18.		$\sim Fa$ $a = a$	$16 \supset D$
	19.		$\sim Fa$ $b = a$ $\sim Fa$ $b = a$	$17 \supset D$
			o × o ×	

The tree has at least one completed open branch, therefore the given sentences are not quantificationally equivalent.

i.

1.	$\sim (((\forall x)Fx \vee (\forall x) \sim Fx) \equiv (\forall y)(Fy \supset y = b))\checkmark$						SM
2.	$(\forall x)Fx \vee (\forall x) \sim Fx\checkmark$		$\sim ((\forall x)Fx \vee (\forall x) \sim Fx)\checkmark$				$1 \sim \equiv D$
3.	$\sim (\forall y)(Fy \supset y = b)\checkmark$		$(\forall y)(Fy \supset y = b)$				$1 \sim \equiv D$
4.	$(\exists y) \sim (Fy \supset y = b)\checkmark$						$3 \sim \forall D$
5.	$\sim (Fa \supset a = b)\checkmark$						$4\ \exists D$
6.	Fa						$5 \sim \supset D$
7.	$\sim a = b$						$5 \sim \supset D$
8.	$(\forall x)Fx$	$(\forall x) \sim Fx$					$2 \vee D$
9.	Fa	$\sim Fa$					$8\ \forall D$
10.	Fb	$\times$					$8\ \forall D$
11.	o		$\sim (\forall x)Fx\checkmark$				$2 \sim \vee D$
12.			$\sim (\forall x) \sim Fx\checkmark$				$2 \sim \vee D$
13.			$(\exists x) \sim Fx\checkmark$				$11 \sim \forall D$
14.			$(\exists x) \sim \sim Fx\checkmark$				$12 \sim \forall D$
15.			$\sim Fa$				$13\ \exists D$
16.			$\sim \sim Fc\checkmark$				$14\ \exists D$
17.			Fc				$16 \sim \sim D$
18.			$Fc \supset c = b\checkmark$				$3\ \forall D$
19.			$\sim Fc$ $\times$	$c = b$			$18 \supset D$
20.				$Fb \supset b = b\checkmark$			$3\ \forall D$
21.				$\sim Fb$	$b = b$		$20 \supset D$
22.				$\sim Fc$			$19, 21 = D$
23.				$\times$	$Fa \supset a = b\checkmark$		$3\ \forall D$
24.					$\sim Fa$	$a = b$	$23 \supset D$
25.					$b = c$	$b = c$	$19, 21 = D$
26.					$c = c$	$c = c$	$19, 25 = D$
27.						$a = c$	$19, 24 = D$
28.						Fa $\times$	$27, 17 = D$
29.					Fb o		$17, 25 = D$

The tree has at least one completed open branch, therefore the given sentences are not quantificationally equivalent.

k.

1.	~ ((∃x) (x = a & x = b) ≡ a = b) ✓		SM
2.	(∃x) (x = a & x = b) ✓	~ (∃x) (x = a & x = b) ✓	1 ~ ≡D
3.	~ a = b	a = b	1 ~ ≡D
4.		(∀x) ~ (x = a & x = b)	2 ~ ∃D
5.		~ (a = a & a = b) ✓	4 ∀D
6.		~ (b = a & b = b)	4 ∀D
7.		~ a = a ~ a = b	5 ~ &D
8.	c = a & c = b ✓	× ×	2 ∃D
9.	c = a		8 &D
10.	c = b		8 &D
11.	~ c = b		3, 9 =D
	×		

The tree is closed. Therefore the sentences ‘(∃x) (x = a & x = b)’ and ‘a = b’ are quantificationally equivalent.

5.a.

1.	a = b & ~ Bab ✓	SM
2.	~ ~ (∀x)Bxx ✓	SM
3.	(∀x)Bxx	2 ~ ~ D
4.	a = b	1 &D
5.	~ Bab	1 &D
6.	Bbb	3 ∀D
7.	Bab	4, 6 =D
	×	

The tree is closed. Therefore the argument is quantificationally valid.

c.

1.	(∀z) (Gz ⊃ (∀y) (Ky ⊃ Hzy))		SM
2.	(Ki & Gj) & i = j ✓		SM
3.	~ Hii		SM
4.	Ki & Gj ✓		2 &D
5.	i = j		2 &D
6.	Ki		4 &D
7.	Gj		4 &D
8.	Gj ⊃ (∀y) (Ky ⊃ Hjy) ✓		1 ∀D
9.	~ Gj	(∀y) (Ky ⊃ Hjy)	8 ⊃D
10.	×	Ki ⊃ Hji ✓	9 ∀D
11.		~ Ki Hji	10 ⊃D
12.		× Hii	5, 11 =D
		×	

The tree is closed. Therefore the argument is quantificationally valid.

e.	1.	a = b		SM
	2.	~ (Ka ∨ ~ Kb)✓		SM
	3.	~ Ka		2 ~ ∀D
	4.	~ ~ Kb✓		2 ~ ∀D
	5.	Kb		4 ~ ~ D
	6.	Ka		1, 5 =D
		×		

The tree is closed. Therefore the argument is quantificationally valid.

g.	1.	(∀x) (x = a ∨ x = b)		SM
	2.	(∃x) (Fxa & Fbx)✓		SM
	3.	~ (∃x)Fxx		SM
	4.	(∀x) ~ Fxx		3 ~ ∃D
	5.	Fca & Fbc✓		2 ∃D
	6.	Fca		5 &D
	7.	Fbc		5 &D
	8.	c = a ∨ c = b✓		1 ∀D
	9.			
	10.	c = a	c = b	8 ∨D
	11.	Fcc		6, 10 =D
	12.		Fcc	7, 10 =D
	13.	~ Fcc	~ Fcc	4 ∀D
		×	×	

The tree is closed. Therefore the argument is quantificationally valid.

i.	1.	(∀x) (∀y) (Fxy ∨ Fyx)		SM
	2.	a = b		SM
	3.	~ (∀x) (Fxa ∨ Fbx)✓		SM
	4.	(∃x) ~ (Fxa ∨ Fbx)✓		3 ~ ∀D
	5.	~ (Fca ∨ Fbc)✓		4 ∃D
	6.	~ Fca		5 ~ ∀D
	7.	~ Fbc		5 ~ ∀D
	8.	(∀y) (Fay ∨ Fya)		1 ∀D
	9.	Fac ∨ Fca✓		8 ∀D
	10.	Fac	Fca	9 ∀D
	11.	~ Fac	×	2, 7 =D
		×		

The tree is closed. Therefore the argument is quantificationally valid.

k. 1.	$(\forall x)(Fx \equiv {\sim} Gx)$			SM
2.	Fa			SM
3.	Gb			SM
4.	$\sim\sim a = b$ ✓			SM
5.	$a = b$			$4 \sim\sim D$
6.	$Fa \equiv {\sim} Ga$ ✓			$1\ \forall D$
7.		Fa	$\sim Fa$	$6 \equiv D$
8.		$\sim Ga$	$\sim\sim Ga$	$6 \equiv D$
9.		Ga	×	3, 5 =D
		×		

The tree is closed. Therefore the argument is quantificationally valid.

m. 1.	$(\forall x)(\forall y)x = y$	SM
2.	$\sim\sim (\exists x)(\exists y)(Fx \,\&\, {\sim} Fy)$ ✓	SM
3.	$(\exists x)(\exists y)(Fx \,\&\, {\sim} Fy)$ ✓	$2 \sim\sim D$
4.	$(\exists y)(Fa \,\&\, {\sim} Fy)$ ✓	$3\ \exists D$
5.	$Fa \,\&\, {\sim} Fb$ ✓	4 $\exists$D
6.	Fa	5 &D
7.	$\sim Fb$	5 &D
8.	$(\forall y)a = y$	$1\ \forall D$
9.	$a = b$	$8\ \forall D$
10.	$\sim Fa$	7, 9 =D
	×	

The tree is closed. Therefore the argument is quantificationally valid.

o. 1.	$(\forall x)(Hx \supset Hf(x))$			SM
2.	$(\exists z) \sim Hf(z)$			SM
3.	$\sim\sim (\forall x)Hx$			SM
4.	$(\forall x)Hx$			$3 \sim\sim D$
5.	$\sim Hf(a)$			$2\ \exists D$
6.	Ha			$4\ \forall D$
7.	$Ha \supset Hf(a)$			$1\ \forall D$
8.		$\sim Ha$	$Hf(a)$	$7 \supset D$
		×	×	

The tree is closed. Therefore the argument is quantificationally valid.

q.

1.	$(\forall x)(\forall y)(Hxy \equiv \sim Hyx)$		SM
2.	$(\exists x)(Hxf(x)\ \&\ \sim Hf(x)x)\checkmark$		SM
3.	$\sim\sim(\forall x)f(x) = x\checkmark$		SM
4.	$(\forall x)f(x) = x$		$3 \sim\sim D$
5.	$Haf(a)\ \&\ \sim Hf(a)a\checkmark$		$2\ \exists D$
6.	$Haf(a)$		5 &D
7.	$\sim Hf(a)a$		5 &D
8.	$(\forall y)(Hay \equiv \sim Hya)$		$1\ \forall D$
9.	$Haa \equiv \sim Haa\checkmark$		$5\ \forall D$
10.	Haa	$\sim Haa$	$9 \equiv D$
11.	$\sim Haa$	$\sim\sim Haa$	$9 \equiv D$
12.	×	Haa	$11 \sim\sim D$
		×	

The tree is closed. Therefore the argument is quantificationally valid.

s.

1.	$(\forall x)[Px \supset (Ox \vee \sim x = f(b))]$			SM
2.	$(\exists x)[(Px\ \&\ \sim Ox)\ \&\ x = f(b)]\checkmark$			SM
3.	$\sim Ob$			SM
4.	$(Pa\ \&\ \sim Oa)\ \&\ a = f(b)\checkmark$			$2\ \exists D$
5.	$Pa\ \&\ \sim Oa\checkmark$			4 &D
6.	$a = f(b)$			4 &D
7.	Pa			5 &D
8.	$\sim Oa$			5 &D
9.	$Pa \supset (Oa \vee \sim a = f(b))\checkmark$			$1\ \forall D$
10.	$Pb \supset (Ob \vee \sim b = f(b))$			$1\ \forall D$
11.	$\sim Pa$	$Oa \vee \sim a = f(b)$		$9 \supset D$
	×			
		Oa	$\sim a = f(b)$	$11 \vee D$
		×	×	

The tree is closed. Therefore the argument is quantificationally valid.

6.a.	1.	$(\forall x)(Fx \supset (\exists y)(Gyx \,\&\, \sim y = x))$		SM
	2.	$(\exists x)Fx \checkmark$		SM
	3.	$\sim (\exists x)(\exists y) \sim x = y \checkmark$		SM
	4.	$(\forall x) \sim (\exists y) \sim x = y$		$3 \sim \exists D$
	5.	Fa		$2\ \exists D$
	6.	$Fa \supset (\exists y)(Gya \,\&\, \sim y = a) \checkmark$		$1\ \forall D$
	7.	$\sim Fa$	$(\exists y)(Gya \,\&\, \sim y = a) \checkmark$	$6 \supset D$
	8.	$\times$	$Gba \,\&\, \sim b = a \checkmark$	$7\ \exists D$
	9.		Gba	$8\ \& D$
	10.		$\sim b = a$	$8\ \& D$
	11.		$\sim (\exists y) \sim a = y \checkmark$	$4\ \forall D$
	12.		$\sim (\exists y) \sim b = y \checkmark$	$4\ \forall D$
	13.		$(\forall y) \sim \sim a = y$	$11 \sim \exists D$
	14.		$(\forall y) \sim \sim b = y$	$12 \sim \exists D$
	15.		$\sim \sim a = a \checkmark$	$13\ \forall D$
	16.		$\sim \sim a = b \checkmark$	$13\ \forall D$
	17.		$\sim \sim b = a \checkmark$	$14\ \forall D$
	18.		$\sim \sim b = b \checkmark$	$14\ \forall D$
	19.		$a = a$	$15 \sim \sim D$
	20.		$a = b$	$16 \sim \sim D$
	21.		$b = a$	$17 \sim \sim D$
	22.		$b = b$	$18 \sim \sim D$
	23.		$\sim b = b$	$10, 21 = D$
			$\times$	

The tree is closed. Therefore the alleged entailment does hold.

c.	1.	$(\forall x)(Fx \supset \sim x = a)$		SM
	2.	$(\exists x)Fx \checkmark$		SM
	3.	$\sim (\exists x)(\exists y) \sim x = y \checkmark$		SM
	4.	Fb		$2\ \exists D$
	5.	$(\forall x) \sim (\exists y) \sim x = y$		$3 \sim \exists D$
	6.	$Fb \supset \sim b = a \checkmark$		$1\ \forall D$
	7.	$\sim Fb$	$\sim b = a$	$6 \supset D$
	8.	$\times$	$\sim (\exists y) \sim a = y \checkmark$	$5\ \forall D$
	9.		$(\forall y) \sim \sim a = y$	$8 \sim \exists D$
	10.		$\sim \sim a = b \checkmark$	$9\ \forall D$
	11.		$a = b$	$10 \sim \sim D$
	12.		$\sim a = a$	$7, 11 = D$
			$\times$	

The tree is closed. Therefore the alleged entailment does hold.

e.	1.	(∃w)(∃z) ~ w = z✓	SM
	2.	(∃w)Hw✓	SM
	3.	~ (∃w) ~ Hw✓	SM
	4.	(∀w) ~ ~ Hw	3 ~ ∃D
	5.	(∃z) ~ a = z✓	1 ∃D
	6.	Hb	2 ∃D
	7.	~ a = c	5 ∃D
	8.	~ ~ Ha✓	4 ∀D
	9.	~ ~ Hb✓	4 ∀D
	10.	~ ~ Hc✓	4 ∀D
	11.	Ha	8 ~ ~ D
	12.	Hb	9 ~ ~ D
	13.	Hc	10 ~ ~ D
		o	

The tree has a completed open branch. Therefore, the alleged entailment does not hold.

g.	1.	(∀x)(∀y)((Fx ≡ Fy) ≡ x = y)							SM
	2.	(∃z)Fz✓							SM
	3.	~ (∃x)(∃y)(~ x = y & (Fx & ~ Fy))✓							SM
	4.	(∀x) ~ (∃y)(~ x = y & (Fx & ~ Fy))							3 ~ ∃D
	5.	Fa							2 ∃D
	6.	~ (∃y)(~ a = y & (Fa & ~ Fy))✓							4 ∀D
	7.	(∀y) ~ (~ a = y & (Fa & ~ Fy))							6 ~ ∃D
	8.	~ (~ a = a & (Fa & ~ Fa))✓							7 ∀D
	9.	(∀y)((Fa ≡ Fy) ≡ a = y)							1 ∀D
	10.	(Fa ≡ Fa) ≡ a = a✓							9 ∀D
	11.	~ ~ a = a✓			~ (Fa & ~ Fa)✓				8 ~ &D
	12.	a = a							11 ~ ~ D
	13.				~ Fa	~ ~ Fa✓			11 ~ &D
	14.				×	Fa			13 ~ ~ D
	15.	Fa ≡ Fa✓		~ (Fa ≡ Fa)✓		Fa ≡ Fa✓		~ (Fa ≡ Fa)✓	10 ≡D
	16.	a = a		~ a = a		a = a		~ a = a	10 ≡D
	17.			×				×	
	18.	Fa	~ Fa			Fa	~ Fa		15 ≡D
	19.	Fa	~ Fa			Fa	~ Fa		15 ≡D
	20.	o	×			o	×		

The tree has at least one completed open branch. Therefore, the alleged entailment does not hold.

i.

1.	$(\forall x)(\forall y)[\sim x = g(y) \supset Gxy]$		SM
2.	$\sim (\exists x)Gax$ ✓		SM
3.	$\sim (\exists x)a = g(x)$ ✓		SM
4.	$(\forall x) \sim a = g(x)$		3 $\sim \exists$D
5.	$\sim a = g(a)$		4 $\forall$D
6.	$(\forall x) \sim Gax$		2 $\sim \exists$D
7.	$(\forall y)[\sim a = g(y) \supset Gay]$		1 $\forall$D
8.	$\sim a = g(a) \supset Gaa$ ✓		7 $\forall$D
9.	$\sim Gaa$		6 $\forall$D
10.	$\sim \sim a = g(a)$	Gaa	8 $\supset$D
11.	$a = g(a)$	×	10 $\sim \sim$ D
	×		

The tree is closed. Therefore the entailment holds.

Section 9.6E

1. a.

1.	$(\forall x)(\forall y)[\sim x = g(y) \supset Gxy]$			SM
2.	$\sim (\exists x)Gax$ ✓			SM
3.	$(\forall x) \sim Gax$			2 ~ED
4.	$(\forall y)[\sim a = g(y) \supset Gay]$			1 $\forall$D
5.	$\sim Gaa$			3 $\forall$D
6.	$\sim a = g(a) \supset Gaa$ ✓			4 $\forall$D
7.	$\sim \sim a = g(a)$ ✓		Gaa	6 $\supset$D
8.	$a = g(a)$		×	7 $\sim \sim$ D
9.	$a = g(a)$	$b = g(a)$		8 CTD
10.	$a = a$	$a = a$		8, 8 =D
11.	o	$a = b$		9, 8 =D
12.		$b = a$		8, 9 =D

This systematic tree has a completed open branches. Therefore, the set being tested is quantificationally consistent.

c.

1.			$(\exists x)(\exists y)Hf(x,y)$ ✓			SM
2.			$\sim (\exists x)Hx$ ✓			SM
3.			$(\forall x) \sim Hx$			$2 \sim \exists D$
4.			$(\exists y)Hf(a,y)$ ✓			$1\ \exists D2$
5.	$Hf(a,a)$			$Hf(a,b)$		$4\ \exists D2$
6.	$a = f(a,a)$	$b = f(a,a)$	$a = f(a,b)$	$b = f(a,b)$	$c = f(a,b)$	5 CTD
7.	$a = a$	$b = b$	$a = a$	$b = b$	$c = c$	6, 6 =D
8.	Ha	Hb	Ha	Hb	Hc	6, 5 =D
9.	$\sim$ Ha	$\sim$ Hb	$\sim$ Ha	$\sim$ Hb	$\sim$ Hc	$3\ \forall D$
	×	×	×	×	×	

This systematic tree is closed. Therefore, the set being tested is quantificationally inconsistent.

e.

1.	$(\forall x)Lxf(x)$		SM
2.	$(\exists y) \sim Lf(y)y$ ✓		SM
3.	$\sim Lf(a)a$		$2\ \exists D2$
4.	$Laf(a)$		$1\ \forall D$
5.	$a = f(a)$	$b = f(a)$	4 CTD
6.	$a = a$	$b = b$	5, 5 =D
7.	$\sim$ Laa	$\sim$ Lba	5, 3 =D
8.	Laa	Lba	5, 4 =D
	×	×	

This systematic tree is closed. Therefore the set being tested is quantificationally inconsistent.

g.

1.	$(\forall x)(Gx \supset \sim Gh(x))$				SM
2.	$(\exists x)(\sim Gx \ \& \sim Gh(x))$ ✓				SM
3.	$\sim Ga \ \& \sim Gh(a)$ ✓				2 ∃D2
4.	$\sim Ga$				4 &D
5.	$\sim Gh(a)$				4 &D
6.	$a = h(a)$		$b = h(a)$		5 CTD
7.	$a = a$		$b = b$		6, 6 =D
8.	$\sim Ga$		$\sim Gb$		6, 5 =D
9.	$Ga \supset \sim Gh(a)$ ✓		$Ga \supset \sim Gh(a)$ ✓		1 ∀D
10.			$Gb \supset \sim Gh(b)$ ✓		1 ∀D
11.	$\sim Ga$	$\sim Gh(a)$	$\sim Ga$ ⋮	$\sim Gh(a)$ ⋮	9 ⊃D

This systematic tree a completed open branches (in fact it has two, the left two). Therefore the set being tested is quantificationally consistent.

2.a.

1.	$\sim (\forall x)(Pf(x) \supset Px)$ ✓		SM
2.	$(\exists x) \sim (Pf(x) \supset Px)$ ✓		1 ~ ∀D
3.	$\sim (Pf(a) \supset Pa)$ ✓		2 ∃D2
4.	$Pf(a)$		3 ⊃D
5.	$\sim Pa$		3 ⊃D
6.	$a = f(a)$	$b = f(a)$	4 CTD
7.	$a = a$	$b = b$	6, 6 =D
8.	Pa	Pb	4, 6 =D
	×	o	

The tree has a completed open branch. Therefore, the sentence being tested is not quantificationally false and the sentence of which it is the negation, '$(\forall x)(Pf(x) \supset Px)$' is not quantificationally true.

c.

1.			$\sim (\exists x)(\forall y)x = g(y)$ ✓			SM
2.			$(\forall x) \sim (\forall y)x = g(y)$			$1 \sim \exists D$
3.			$\sim (\forall y)a = g(y)$ ✓			$2\ \forall D$
4.			$(\exists y) \sim a = g(y)$ ✓			$3 \sim \forall D$
5.		$\sim a = g(a)$		$\sim a = g(b)$		$4\ \exists D2$
6.	$a = g(a)$	$b = g(a)$	$a = g(b)$	$b = g(b)$	$c = g(b)$	5 CTD
7.	×	$b = b$	×	$b = b$	$c = c$	6, 6 =D
8.		$\sim a = b$		$\sim a = b$	$\sim a = c$	5, 6 =D
9.		$\sim (\forall y)b = g(y)$ ✓		$\sim (\forall y)b = g(y)$ ✓	$\sim (\forall y)b = g(y)$	$2\ \forall D$
10.					$\sim (\forall y)c = g(y)$	$2\ \forall D$
11.		$(\exists y) \sim b = g(y)$ ✓		$(\exists y) \sim b = g(y)$	$(\exists y) \sim b = g(y)$	$9 \sim \forall D$
12.					$(\exists y) \sim c = g(y)$	$10 \sim \forall D$
13.	$\sim b = g(a)$	$\sim b = g(b)$	$\sim b = g(c)$	incomplete	incomplete	$11\ \exists D2$
	×					
14.	$a = g(b)$	$b = g(b)$	$c = g(b)$	incomplete		13 CTD
15.	$a = a$	$b = b$	$c = c$			14, 14 =D
16.	$\sim b = a$	$\sim b = b$	$\sim b = c$			13, 14 =D
	o	×	⋮			

If we were to complete the indicated missing work, we would have a systematic tree with at least one completed open branch (the left most branch). Therefore, the sentence being tested is not quantificationally false and the sentence of which it is a negation, '$(\exists x)(\forall y)x = g(y)$' is not quantificationally true.

e.

1.	$\sim (\forall x)(\forall y)(Dh(x,y) \supset Dh(y,x))$ ✓							SM
2.	$(\exists x) \sim (\forall y)(Dh(x,y) \supset Dh(y,x))$ ✓							$1 \sim \forall D$
3.	$\sim (\forall y)(Dh(a,y) \supset Dh(y,a))$ ✓							$2\ \exists D2$
4.	$(\exists y) \sim (Dh(a,y) \supset Dh(y,a))$ ✓							$3 \sim \forall D$
5.	$\sim (Dh(a,a) \supset Dh(a,a))$ ✓			$\sim (Dh(a,b) \supset Dh(b,a))$ ✓				$4\ \exists D2$
6.	$Dh(a,a)$			$Dh(a,b)$				$5 \sim \supset D$
7.	$\sim Dh(a,a)$			$\sim Dh(b,a)$				$5 \sim \supset D$
	×							
8.	$a = h(a,b)$			$b = h(a,b)$			$c = h(a,b)$	6 CTD
9.	$a = h(b,a)$	$b = h(b,a)$	$c = h(b,a)$	$a = h(b,a)$	$b = h(b,a)$	$c = h(b,a)$	incomplete	7 CTD
10.	$a = a$	$b = b$	$c = c$	$a = a$	$b = b$	$c = c$		9, 9 =D
11.	Da	Da	Da	Db	Db	Db		6, 8 =D
12.	$\sim$ Da	$\sim$ Db	$\sim$ Dc	$\sim$ Da	$\sim$ Db	$\sim$ Dc		7, 9 =D
	×				×			

If we were to complete the application of CTD and =D on the far right branch we would have a systematic tree with at least one completed open branch. Therefore, the sentence being tested is not quantificationally false, and the sentence of which it is the negation, '$(\forall x)(\forall y)(Dh(x,y) \supset Dh(y,x))$' is not quantificationally true.

3. a.

1.	$\sim (\forall x)(\exists y)y = f(f(x))$ ✓				SM
2.	$(\exists x) \sim (\exists y)y = f(f(x))$ ✓				$1 \sim \forall D$
3.	$\sim (\exists y)y = f(f(a))$ ✓				$2\ \exists D2$
4.	$(\forall y) \sim y = f(f(a))$				$3 \sim \exists D$
5.	$\sim a = f(f(a))$				$4\ \forall D$
6.	$a = f(f(a))$	$b = f(f(a))$			5 CTD
	×				
7.		$a = f(a)$	$b = f(a)$	$c = f(a)$	6 CTD
8.		$b = b$	$b = b$	$b = b$	6, 6 =D
9.		$a = a$		$c = c$	7, 7 =D
10.		$\sim a = f(a)$	$\sim a = f(b)$	$\sim a = f(c)$	7, 5 =D
11.		×	$b = f(b)$	$b = f(c)$	7, 6 =D
12.				$\sim a = b$	10, 11 =D
13.			$\sim b = f(f(a))$	$\sim b = f(f(a))$	$4\ \forall D$
14.				$\sim c = f(f(a))$	$4\ \forall D$
15.			$\sim b = f(b)$	$\sim b = f(c)$	7, 13 =D
			×	×	

The tree is closed. Therefore '$\sim (\forall x)(\exists y)y = f(f(x))$' is quantificationally false and '$(\forall x)(\exists y)y = f(f(x))$' is quantificationally true.

c. 1.	$\sim [(\forall x)Lf(x) \supset (\forall x)Lf(f(x))]\checkmark$	SM
2.	$(\forall x)Lf(x)$	$1 \sim \supset D$
3.	$\sim (\forall x)Lf(f(x))\checkmark$	$1 \sim \supset D$
4.	$(\exists x) \sim Lf(f(x))\checkmark$	$3 \sim \supset D$
5.	$\sim Lf(f(a))$	$4\ \exists D2$
6.	$Lf(a)$	$2\ \forall D$
7.	$\sim Lf(f(a))$	$2\ \forall D$
	$\times$	

4.a. 1.		$\sim (\exists y) = f(f(x)\checkmark$			SM
4.		$(\forall x) \sim f(x) = x$			$1 \sim \exists D$
3.		$\sim a = f(f(a))$			$4\ \forall D$
4.	$a = f(a)$		$b = f(a)$		3 CTD
5.	$\sim a = a$		$\sim b = a$		3, 4 =D
6.	$\times$		$b = b$		4, 4 =D
7.			$\sim f(b) = b$		$2\ \forall D$
8.		$a = f(b)$	$b = f(b)$	$c = f(b)$	7 CTD
9.		$\sim a = b$	$\sim b = b$	$\sim c = b$	7, 8 =D
10.		$a = a$	$\times$	$c = c$	8, 8 =D
		o			

. 1.	$\sim (\exists x)f(x) = x\checkmark$		SM
2.	$f(a) = a$		$1\ \exists D2$
3.	$a = f(a)$	$b = f(b)$	2 CTD
4.	$a = a$	$b = b$	3, 3 =D
	o		

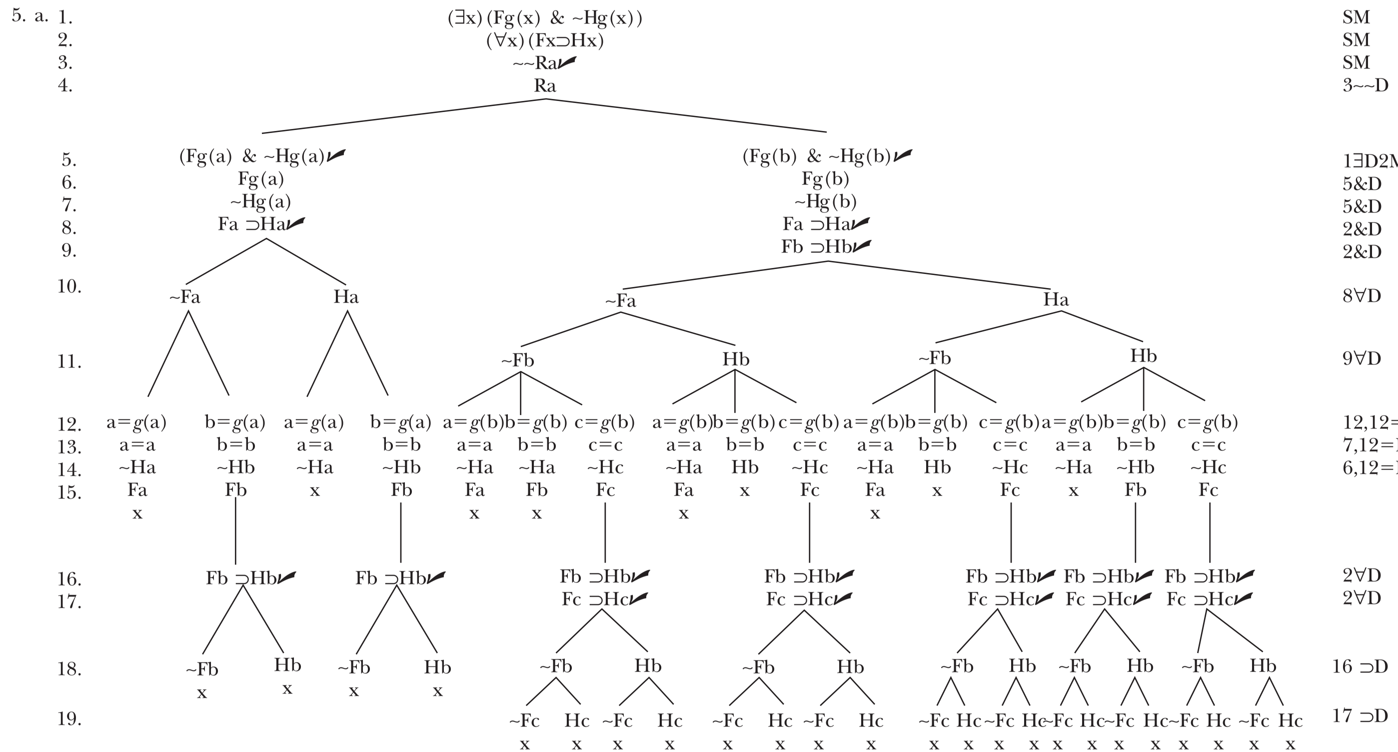
The tree for the negation of the given sentence has at least one completed open branch, and the tree for the given sentence has at least one completed open branch. Therefore the given sentence is quantificationally indeterminate.

c. 1.		$(\forall x)(\exists y)y = f(f(x))$			SM
2.		$(\exists y)y = f(f(a))\checkmark$			$1\ \forall D$
3.	$a = f(f(a))$		$b = f(f(a))$		$2\ \exists D2$
4.	$a = f(a)$	$b = f(a)$	$a = f(a)$	$b = f(a)$	3 CTD
5.	$a = a$	$b = b$	$a = a$	$b = b$	4, 4 =D
	o	$\vdots$	$\vdots$	$\vdots$	

The tree has one completed open branch. Therefore the sentence '$(\forall x)(\exists y)y = f(f(x))$' is not quantificationally false.

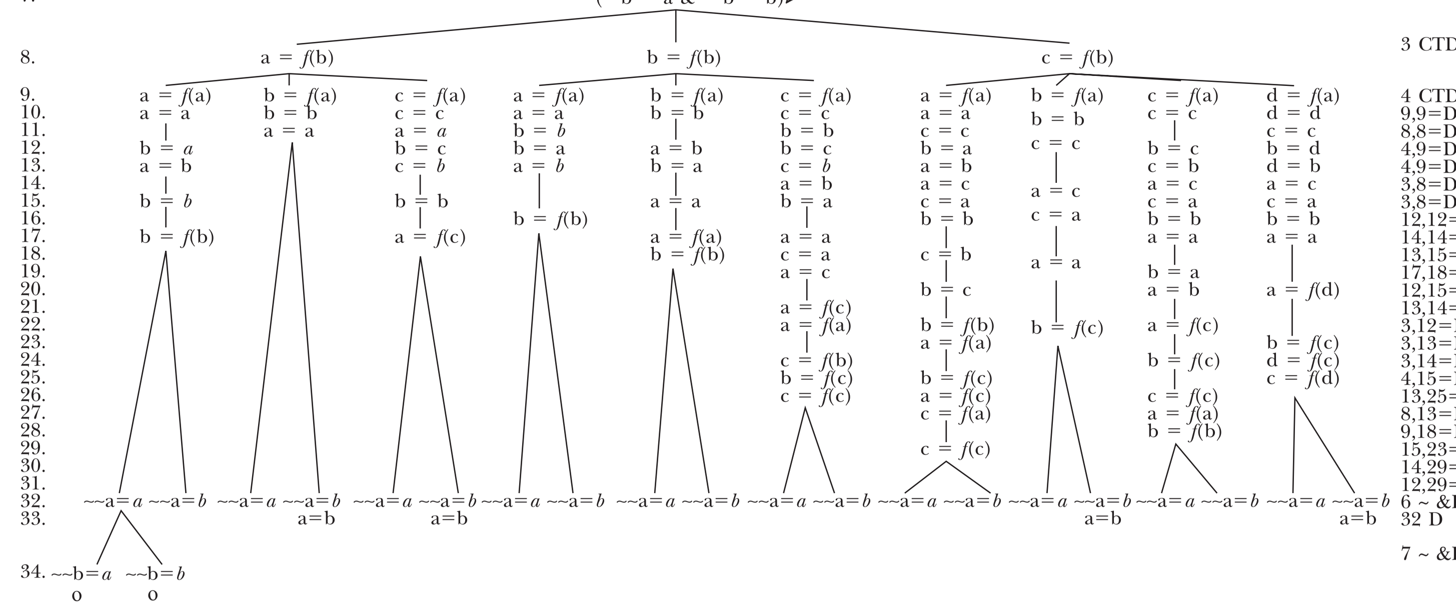
1.			$\sim (\forall x)(\exists y)y = f(f(x))$ ✓		SM
2.			$(\exists x) \sim (\exists y)y = f(f(x))$ ✓		$1 \sim \forall D$
3.			$\sim (\exists y)y = f(f(a))$ ✓		$2\ \exists D2$
4.			$(\forall y) \sim y = f(f(a))$		$3 \sim \exists D$
5.			$\sim a = f(f(a))$		$4\ \forall D$
6.	$a = f(a)$		$b = f(a)$		5 CTD
7.	$a = a$		$b = b$		6, 6 =D
8.	$\sim a = f(a)$		$\sim a = f(b)$		6, 5 =D
9.	×		$\sim b = f(f(a))$		$4\ \forall D$
10.		$a = f(b)$	$b = f(b)$	$c = f(b)$	8 CTD
11.		$a = a$		$c = c$	10, 10 =D
12.		$\sim a = a$	$\sim a = b$	$\sim a = c$	10, 8 =D
13.		×	$\sim b = f(b)$	$\sim b = f(b)$	6, 9 =D
14.			$\sim b = b$	$\sim b = c$	10, 13 =D
15.			×	$\sim c = f(f(a))$	$4\ \forall D$
16.				$\sim c = f(b)$	6, 15 =D
17.				$\sim c = c$	10, 16 =D
				×	

The tree is closed. Therefore '$\sim (\forall x)(\exists y)y = f(f(x))$' is quantificationally false and '$(\forall x)(\exists y)y = f(f(x))$' is quantificationally true.



The tree for the premises and the negation of the conclusion is closed. Therefore, the argument being tested is quantificationally valid.

c.1.	a = g(a) & b = f(a)	SM
2.	~ (∃x) ~ (~ x = a & ~ x = b)	SM
3.	a = f(b)	1 & D
4.	b = f(a)	1 & D
5.	(∀x) ~ (~ x = a & ~ x = b)	2 ~ ∃D
6.	~ (~ a = a & ~ a = b)	5 ∀D
7.	~ (~ b = a & ~ b = b)	5 ∀D



The tree has two completed open branches. Therefore, the argument is not quantificationally valid.

6. a.

1.				$\sim [(\forall x)(\exists y)y = f(x) \equiv (\forall x)(\exists y)x = f(y)]$ ✓				SM
2.	$(\forall x)(\exists y)y = f(x)$					$\sim (\forall x)(\exists y)y = f(x)$ ✓		$1 \sim \equiv D$
3.	$\sim (\forall x)(\exists y)x = f(y)$ ✓					$(\forall x)(\exists y)x = f(y)$		$1 \sim \equiv D$
4.	$(\exists x) \sim (\exists y)x = f(y)$ ✓							$3 \sim \forall D$
5.	$\sim (\exists y)a = f(y)$ ✓							$4\ \exists D2$
6.	$(\forall y) \sim a = f(y)$							$5 \sim \exists D$
7.						$(\exists x) \sim (\exists y)y = f(x)$ ✓		$2 \sim \forall D$
8.						$\sim (\exists y)y = f(a)$ ✓		$7\ \exists D2$
9.						$(\forall y) \sim y = f(a)$		$9 \sim \exists D$
10.						$\sim f(a) = f(a)$		$10\ \forall D$
11.	$\sim a = f(a)$					x		$6\ \forall D$
12.	$(\exists y)y = f(a)$ ✓							$2\ \forall D$
13.	$a = f(a)$	$b = f(a)$						$12\ \exists D2$
14.	x	$\sim a = f(b)$						$6\ \forall D$
15.		$(\exists y)y = f(b)$ ✓						$2\ \forall D$
16.	$a = f(b)$	$b = f(b)$			$c = f(b)$			$15\ \exists D2$
	x							
17.	$a = f(a)$	$b = f(a)$	$c = f(a)$	$a = f(a)$	$b = f(a)$	$c = f(a)$	$d = f(a)$	13 CTD
18.	x	$b = b$	$c = c$	x	$b = b$	$c = c$	$d = d$	$17, 17 = D$
19.		$\sim a = b$	$\sim a = c$		$\sim a = b$	$\sim a = c$	$\sim a = d$	$11, 17 = D$
		o						

The tree has a completed open branch. Therefore the sentences are not quantificationally equivalent.

c. 1.	~[(∃x)x=x=(∃x)x=*f*(x)]✓				SM
2.	(∃x)x=x			~(∃x)x=x✓	1~ ≡D
3.	~(∃x)x=*f*(x)✓			(∃x)x=*f*(x)	1~ ≡D
4.	(∀x)~x=*f*(x)				2~ ∃D
5.				(∀x)~x=x	2∃D2
6.	a=a				3∃D2
7.				a=*f*(a)	4∀D
8.	~a=f(a)				5∀D
9.				~a=a	
				×	
10.	a=f(a)	b=*f*(a)			8CTD
11.	×	b=b			10,10=D
12.		~a=b			8,10=D
13.		~b=*f*(b)			4∀D
14.	a=*f*(b)	b=*f*(b)	c=*f*(b)		13CTD
15.	a=a	×	c=c		14,14=D
16.	~b=a		~b=c		13,14=D
	o				

The systematic tree has at least one completed open branch. Therefore the sentences are not quantificationally equivalent.

7. a. 1.	(∀x)(∀y)x = *g*(x,y)	SM
2.	~ (∀x)x = *g*(x,x)✓	SM
3.	(∃x) ~ x = *g*(x,x)✓	2 ~ ∀D
4.	~ a = *g*(a,a)	3 ∃D2
5.	(∀y)a = *g*(a,y)	1 ∀D
6.	a = *g*(a,a)	1 ∀D
	×	

This tree is closed. Therefore, the alleged entailment does hold.

c. 1.			$\sim (\forall x)x = f(f(x))$		SM
2.			$\sim (\forall x)x = f(x)$ ✓		SM
3.			$\sim (\exists x) \sim x = f(a)$ ✓		$2 \sim \forall D$
4.			$\sim a = f(a)$		$3\ \exists D$
5.			$a = f(f(a))$		$1\ \forall D$
6.	$a = f(a)$		$b = f(a)$		4 CTD
7.	×		$\sim a = b$		4, 6 =D
8.			$a = f(b)$		5, 6 =D
9.			$b = b$		6, 6 =D
10.			$a = a$		8, 8 =D
11.			$b = f(f(b))$		$1\ \forall D$
10.		$a = f(b)$	$b = f(b)$	$c = f(b)$	8 CTD
		o			

The tree has at least one completed one branch. Therefore the alleged entailment does not hold.