

CHAPTER SEVEN

Section 7.2E

- 1.a. 'The President' is a singular term, 'Democrat' is not
x is a Democrat
('w' or 'y' or 'z' may be used in place of 'x')
- c. 'Sarah' and 'Smith College' are the singular terms
x attends Smith College
Sarah attends x
x attends y
- e. The singular terms are 'Charles' and 'Rita'
w and Rita are brother and sister
Charles and w are brother and sister
w and z are brother and sister
- g. The singular terms are '2', '4', and '8'
x times 4 is 8
2 times x is 8
2 times 4 is y
x times y is 8
x times 4 is y
2 times x is y
x times y is z
- i. The singular terms are '0', '0', and '0'
z plus 0 is 0
0 plus z is 0
0 plus 0 is z
w plus y is 0
w plus 0 is y
0 plus w is y
w plus y is z
2. Herman is larger than Herman.
Herman is larger than Juan.
Herman is larger than Antonio.
Juan is larger than Herman.
Juan is larger than Juan.
Juan is larger than Antonio.
Antonio is larger than Herman.
Antonio is larger than Juan.
Antonio is larger than Antonio.

Herman is to the right of Herman.
Herman is to the right of Juan.
Herman is to the right of Antonio.
Juan is to the right of Herman.
Juan is to the right of Juan.
Juan is to the right of Antonio.
Antonio is to the right of Herman.
Antonio is to the right of Juan.
Antonio is to the right of Antonio.

Herman is larger than Herman but smaller than Herman.
Herman is larger than Herman but smaller than Juan.
Herman is larger than Herman but smaller than Antonio.
Herman is larger than Juan but smaller than Herman.
Herman is larger than Juan but smaller than Juan.
Herman is larger than Juan but smaller than Antonio.
Herman is larger than Antonio but smaller than Herman.
Herman is larger than Antonio but smaller than Juan.
Herman is larger than Antonio but smaller than Antonio.

Juan is larger than Herman but smaller than Herman.
Juan is larger than Herman but smaller than Juan.
Juan is larger than Herman but smaller than Antonio.
Juan is larger than Juan but smaller than Herman.
Juan is larger than Juan but smaller than Juan.
Juan is larger than Juan but smaller than Antonio.
Juan is larger than Antonio but smaller than Herman.
Juan is larger than Antonio but smaller than Juan.
Juan is larger than Antonio but smaller than Antonio.

Antonio is larger than Herman but smaller than Herman.
Antonio is larger than Herman but smaller than Juan.
Antonio is larger than Herman but smaller than Antonio.
Antonio is larger than Juan but smaller than Herman.
Antonio is larger than Juan but smaller than Juan.
Antonio is larger than Juan but smaller than Antonio.
Antonio is larger than Antonio but smaller than Herman.
Antonio is larger than Antonio but smaller than Juan.
Antonio is larger than Antonio but smaller than Antonio.

EXERCISES 7.3E

1. The *PL* analogs of the sentences of English, in the same order given in the *Solution Manual* answers to exercise 7.2E 2, are

Lhh
Lhj
Lha
Ljh
Ljj
Lja
Lah
Laj
Laa

Rhh
Rhj
Rha
Rjh
Rjj
Rja
Rah
Raj
Raa

Shhh
Shhj
Shha
Shjh
Shjj
Shja
Shah
Shaj
Shaa

Sjhh
Sjhj
Sjha
Sjjh
Sjjj
Sjja
Sjah
Sjaj
Sjaa

Sahh
Sahj
Saha
Sajh
Sajj
Saja
Saah
Saaj
Saaa

2. a. Bai
c. Bbn
e. Beh
g. (Aph & Ahn) & Ank
i. Aih \equiv Aip
k. $([(Lap \& Lbp) \& (Lcp \& Ldp)] \& Lep) \& \sim ([(Bap \vee Bbp) \vee (Bcp \vee Bdp)] \vee Bep)$
m. (Tda & Tdb) & (Tdc & Tde)
o. $\sim ([(Tab \vee Tac) \vee (Tad \vee Tae)] \vee Taa) \& [(Lab \& Lac) \& (Lad \& Lae)]$
3. a. (Ia & Ba) & \sim Ra
c. (Bd & Rd) & Id
e. Ib \supset (Id & Ia)
g. Lab & Dac
i. $\sim (Lca \vee Dca) \& (Lcd \& Dcd)$
k. Acb \equiv (Sbc & Rb)
m. (Sdc & Sca) \supset Sda
o. (Lcb & Lba) \supset (Dca & Sca)
q. Rd & $\sim [Ra \vee (Rb \vee Rc)]$
4. a. UD: Margaret, Todd, Charles, and Sarah
Gx: x is good at skateboarding
Lx: x likes skateboarding
Hx: x wears headgear
Kx: x wears knee pads
Rxy: x is more reckless than y (at skateboarding)
Sxy: x is more skillful than y (at skateboarding)
c: Charles
m: Margaret
s: Sarah
t: Todd

$(Lm \ \& \ Lt) \ \& \ \sim \ (Gm \ \vee \ Gt)$
 $Gc \ \& \ \sim \ Lc$
 $Gs \ \& \ Ls$
 $[(Hm \ \& \ Ht) \ \& \ (Hc \ \& \ Hs)] \ \& \ [(Kc \ \& \ Ks) \ \& \ \sim \ (Km \ \vee \ Kt)]$
 $[(Rsm \ \& \ Rst) \ \& \ Rsc] \ \& \ [(Scs \ \& \ Scm) \ \& \ Sct]$

Note: it may be tempting to use a two-place predicate to symbolize being good at skateboarding, for example, ‘Gxy’, and another two-place predicate to symbolize liking skateboarding. So too we might use two-place predicates to symbolize wearing headgear and wearing kneepads. Doing so would require including skateboarding, headgear, and knee pads in the universe of discourse. But things are now a little murky. Skateboarding is more of an activity than a thing (although activities are often the “topics of conversation” as when we say that some people like, for example, hiking, skiing, and canoeing while others don’t). And while we might include all headgear and kneepads in our universe of discourse, we do not know which ones the characters in our passage wear, so we would be hard pressed to name the favored items.

Moreover, here there is no need to invoke these two-place predicates because here we are not asked to investigate logical relations that can only be expressed with two-place predicates. The case would be different if the passage included the sentence ‘If Sarah is good at anything she is good at sailing’ and we were asked to show that it follows from the passage that Sarah is good at sailing. (On the revised scenario we are told that Sarah is good at skateboarding, and that if she is good at anything—she is, skateboarding—she is good at sailing. So she is good at sailing. Here we are treating skateboarding as *something*, something Sarah is good at. But we will leave these complexities until we have fully developed the language *PL*.)

c. One appropriate symbolization key is

UD: Andrew, Christopher, Amanda
Hz: z is a hiker
Mz: z is a mountain climber
Kz: z is a kayaker
Sz: z is a swimmer
Lzw: z likes w
Nzw: z is nuts about w
a: Andrew
c: Christopher
m: Amanda

$(Ha \ \& \ Hc) \ \& \ \sim \ (Ma \ \vee \ Mc)$
 $(Hm \ \& \ Mm) \ \& \ Km$
 $(Ka \ \vee \ Kc) \ \& \ \sim \ (Ka \ \& \ Kc)$
 $\sim \ [(Sa \ \vee \ Sc) \ \vee \ Sm]$
 $((Lac \ \& \ Lca) \ \& \ [(Lam \ \& \ Lma) \ \& \ (Lmc \ \& \ Lcm)]) \ \& \ (Nma \ \& \ Nam)$

Section 7.4E

- 1.a. $(\forall z)Bz$
 c. $\sim (\exists x)Bx$
 e. $(\exists x)Bx \ \& \ (\exists x)Rx$
 g. $(\exists z)Rz \supset (\exists z)Bz$
 i. $(\forall y)By \equiv \sim (\exists y)Ry$
- 2.a. $(\exists x)Ox \ \& \ (\exists x)Ex$
 c. $\sim (\exists x)Lxa$
 e. $(\forall x)Gx$
 g. $(\exists x)(Px \ \& \ Ex)$
 i. $(\forall y)[(Py \ \& \ Lby) \supset Ey]$
 k. $(\exists y)(Lby \ \& \ Lyc)$
- 3.a. $Pj \supset (\forall x)Px$
 c. $(\exists y)Py \supset (Pj \ \& \ Pr)$
 e. $\sim Pr \supset \sim (\exists x)Px$
 g. $(Pj \supset Pr) \ \& \ (Pr \supset (\forall x)Px)$
 i. $(\forall y)Sy \ \& \ \sim (\forall y)Py$
 k. $(\forall x)Sx \supset (\exists y)Py$

Section 7.5E

- 1.a. A formula but not a sentence (an open sentence): the 'z' in 'Zz' is free.
 c. A formula and a sentence.
 e. A formula but not a sentence (an open sentence): the 'x' in 'Fxz' is free.
 g. A formula and a sentence.
 i. Not a formula. ' $\sim (\exists x)$ ' is an expression of *SL*, but ' $(\sim \exists x)$ ' is not.
 k. Not a formula. Since there is no 'y' in 'Lxx', ' $(\exists y)Lxx$ ' is not a formula. Hence, neither is ' $(\exists x)(\exists y)Lxx$ '.
 m. A formula and a sentence.
 o. A formula but not a sentence (an open sentence): 'w' in 'Fw' is free.
- 2.a. A sentence. The subformulas are

$(\exists x)(\forall y)Byx$
 $(\forall y)Byx$
 Byx

$(\exists x)$
 $(\forall y)$
 None

c. Not a sentence. The 'x' in '(Bg \supset Fx)' is free. The subformulas are

$(\forall x)(\sim Fx \ \& \ Gx) \equiv (Bg \supset Fx)$	\equiv
$(\forall x)(\sim Fx \ \& \ Gx)$	$(\forall x)$
$Bg \supset Fx$	\supset
$\sim Fx \ \& \ Gx$	$\&$
$\sim Fx$	\sim
Gx	None
Bg	None
Fx	None

e. Sentence. The subformulas are

$\sim (\exists x)Px \ \& \ Rab$	$\&$
$\sim (\exists x)Px$	\sim
Rab	None
$(\exists x)Px$	$(\exists x)$
Px	None

g. Sentence. The subformulas are

$\sim [\sim (\forall x)Fx \equiv (\exists w) \sim Gw] \supset Maa$	\supset
$\sim [\sim (\forall x)Fx \equiv (\exists w) \sim Gw]$	\sim
Maa	None
$\sim (\forall x)Fx \equiv (\exists w) \sim Gw$	\equiv
$\sim (\forall x)Fx$	\sim
$(\exists w) \sim Gw$	$(\exists w)$
$(\forall x)Fx$	$(\forall x)$
Fx	None
$\sim Gw$	\sim
Gw	None

i. Sentence. The subformulas are

$\sim \sim \sim (\exists x)(\forall z)(Gxaz \vee \sim Hazb)$	\sim
$\sim \sim (\exists x)(\forall z)(Gxaz \vee \sim Hazb)$	\sim
$\sim (\exists x)(\forall z)(Gxaz \vee \sim Hazb)$	\sim
$(\exists x)(\forall z)(Gxaz \vee \sim Hazb)$	$(\exists x)$
$(\forall z)(Gxaz \vee \sim Hazb)$	$(\forall z)$
$Gxaz \vee \sim Hazb$	\vee
$Gxaz$	None
$\sim Hazb$	\sim
$Hazb$	None

k. Sentence. The subformulas are

$(\exists x)[Fx \supset (\forall w)(\sim Gx \supset \sim Hwx)]$	$(\exists x)$
$Fx \supset (\forall w)(\sim Gx \supset \sim Hwx)$	\supset
Fx	None
$(\forall w)(\sim Gx \supset \sim Hwx)$	$(\forall w)$
$\sim Gx \supset \sim Hwx$	\supset
$\sim Gx$	\sim
$\sim Hwx$	\sim
Gx	None
Hwx	None

m. A sentence. The subformulas are

$(Hb \vee Fa) \equiv (\exists z)(\sim Fz \ \& \ Gza)$	\equiv
$Hb \vee Fa$	\vee
$(\exists z)(\sim Fz \ \& \ Gza)$	$(\exists z)$
Hb	None
Fa	None
$\sim Fz \ \& \ Gza$	$\&$
$\sim Fz$	\sim
Gza	None
Fz	None

3.a. $(\forall x)(Fx \supset Ga)$	Quantified
c. $\sim (\forall x)(Fx \supset Ga)$	Truth-functional
e. $\sim (\exists x)Hx$	Truth-functional
g. $(\forall x)(Fx \equiv (\exists w)Gw)$	Quantified
i. $(\exists w)(Pw \supset (\forall y)(Hy \equiv \sim Kyw))$	Quantified
k. $\sim [(\exists w)(Jw \vee Nw) \vee (\exists w)(Mw \vee Lw)]$	Truth-functional
m. $(\forall z)Gza \supset (\exists z)Fz$	Truth-functional
o. $(\exists z) \sim Hza$	Quantified
q. $(\forall x) \sim Fx \equiv (\forall z) \sim Hza$	Truth-functional

4.a. $Maa \ \& \ Fa$

c. $\sim (Ca \equiv \sim Ca)$
e. $(Fa \ \& \ \sim Gb) \supset (Bab \vee Bba)$
g. $\sim (\exists z)Naz \equiv (\forall w)(Mww \ \& \ Naw)$
i. $Fab \equiv Gba$
k. $\sim (\exists y)(Hay \ \& \ Hya)$
m. $(\forall y)[(Hay \ \& \ Hya) \supset (\exists z)Gza]$

5.a. $(\forall y)Ray \supset Byy$	No
c. $(\forall y)(Rwy \supset Byy)$	No
e. $(\forall y)(Ryy \supset Byy)$	No
g. $(Ray \supset Byy)$	No
i. $Rab \supset Bbb$	No
6.a. $(\forall y) \sim Ray \equiv Paa$	Yes
c. $(\forall y) \sim Ray \equiv Pba$	No
e. $(\forall y)(\sim Ryy \equiv Paa)$	No
g. $(\forall y) \sim Raw \equiv Paa$	No

Section 7.6E

1.a. A-sentence	$(\forall y)(Py \supset Cy)$
c. O-sentence	$(\exists w)(Dw \ \& \ \sim Sw)$
e. I-sentence	$(\exists z)(Nz \ \& \ Bz)$
g. E-sentence	$(\forall x)(Px \supset \sim Sx)$
i. A-sentence	$(\forall w)(Pw \supset Mw)$
k. A-sentence	$(\forall y)(Sy \supset Cy)$
m. E-sentence	$(\forall y)(Ky \supset \sim Sy)$
o. E-sentence	$(\forall y)(Qy \supset \sim Zy)$
2.a. $(\forall y)(By \supset Ly)$	
c. $(\forall z)(Rz \supset \sim Lz)$	
e. $(\exists x)Bx \ \& \ (\exists x)Rx$	
g. $[(\exists z)Bz \ \& \ (\exists z)Rz] \ \& \ \sim (\exists z)(Bz \ \& \ Rz)$	
i. $(\exists y)By \ \& \ [(\exists y)Sy \ \& \ (\exists y)Ly]$	
k. $(\forall w)(Cw \supset Rw) \ \& \ \sim (\forall w)(Rw \supset Cw)$	
m. $(\forall y)Ry \vee [(\forall y)By \vee (\forall y)Gy]$	
o. $(\exists w)(Rw \ \& \ Sw) \ \& \ (\exists w)(Rw \ \& \ \sim Sw)$	
q. $(\exists x)Ox \ \& \ (\forall y)(Ly \supset \sim Oy)$	

3.a. An I-sentence and the corresponding O-sentence of *PL* can both be true. Consider the English sentences ‘Some positive integers are even’ and ‘Some positive integers are not even’. Where the UD is positive integers and ‘Ex’ is interpreted as ‘x is even’, these can be symbolized as ‘ $(\exists x)Ex$ ’ and ‘ $(\exists x) \sim Ex$ ’, respectively, and both sentences of *PL* are true.

An I-sentence and an O-sentence can also both be false. Consider ‘Some tiggers are fast’ and ‘Some tiggers are not fast’. Where the UD is mammals, ‘Tx’ is interpreted as ‘x is a tigger’ and ‘Fx’ as ‘x is fast’, these become, respectively, ‘ $(\exists x)(Tx \ \& \ Fx)$ ’ and ‘ $(\exists x)(Tx \ \& \ \sim Fx)$ ’. As there are no tiggers, both sentences of *PL* are false. Note, however, that there cannot be an I-sentence and a corresponding O-sentence of the sorts $(\exists x)\mathbf{A}$ and $(\exists x) \sim \mathbf{A}$, where \mathbf{A} is an atomic formula and both the I-sentence and the O-sentence are false. For however \mathbf{A} is interpreted, either there is something that satisfies it, or there is not. In the first instance $(\exists x)\mathbf{A}$ is true, in the second $(\exists x) \sim \mathbf{A}$ is true.

Section 7.7E

- 1.a. $(\forall z)(Pz \supset Hz)$
 c. $(\exists z)(Pz \ \& \ Hz)$
 e. $(\forall w)[(Hw \ \& \ Pw) \supset \sim Iw]$
 g. $\sim (\forall x)[(Px \vee Ix) \supset Hx]$
 i. $(\forall y)[(Iy \ \& \ Hy) \supset Ry]$
 k. $(\exists z)Iz \supset Ih$
 m. $(\exists w)Iw \supset (\forall x)(Rx \supset Ix)$
 o. $\sim (\exists y)[Hy \ \& \ (Py \ \& \ Iy)]$
 q. $(\forall z)(Pz \supset Iz) \supset \sim (\exists z)(Pz \ \& \ Hz)$
 s. $(\forall w)(Rw \supset [(Lw \ \& \ Iw) \ \& \ \sim Hw])$
- 2.a. $(\forall w)(Lw \supset Aw)$
 c. $(\forall x)(Lx \supset Fx) \ \& \ (\forall x)(Tx \supset \sim Fx)$
 e. $(\exists y)[(Fy \ \& \ Ly) \ \& \ Cdy]$
 g. $(\forall z)[(Lz \vee Tz) \supset Fz]$
 i. $(\exists w)(Tw \ \& \ Fw) \ \& \ \sim (\forall w)(Tw \supset Fw)$
 k. $(\forall x)[(Lx \ \& \ Cbx) \supset (Ax \ \& \ \sim Fx)]$
 m. $(\exists z)(Lz \ \& \ Fz) \supset (\forall w)(Tw \supset Fw)$
 o. $\sim Fb \ \& \ Bb$
- 3.a. $(\forall x)(Ex \supset Yx)$
 c. $(\exists y)(Ey \ \& \ Yy) \ \& \ \sim (\forall y)(Ey \supset Yy)$
 e. $(\exists z)(Ez \ \& \ Yz) \supset (\forall x)(Lx \supset Yx)$
 g. $(\forall w)[(Ew \ \& \ Sw) \supset Yw]$
 i. $(\forall w)[(Lw \ \& \ Ew) \supset (Yw \ \& \ Iw)]$
 k. $(\forall x)[(Ex \vee Lx) \supset (Yx \supset Ix)]$
 m. $\sim (\exists z)[(Pz \ \& \ \sim Iz) \ \& \ Yz]$
 o. $(\forall x)[(Ex \ \& \ Rxx) \supset Yx]$
 q. $(\forall x)[(Ex \vee Lx) \ \& \ (Rx \vee Yx)] \supset Rxx$
 s. $(\forall z)[(Yz \ \& \ (Lz \ \& \ Ez))] \supset Rzz$
- 4.a. $(\forall x)[Px \supset (Ux \ \& \ Ox)]$
 c. $(\forall z)[Az \supset \sim (Oz \vee Uz)]$
 e. $(\forall w)(Ow \equiv Uw)$
 g. $(\exists y)(Py \ \& \ Uy) \ \& \ (\forall y)[(Py \ \& \ Ay) \supset \sim Uy]$
 i. $(\exists z)[Pz \ \& \ (Oz \ \& \ Uz)] \ \& \ (\forall x)[Sx \supset (Ox \ \& \ Ux)]$
 k. $((\exists x)(Sx \ \& \ Ux) \ \& \ (\exists x)(Px \ \& \ Ux)) \ \& \ \sim (\exists x)(Ax \ \& \ Ux)$
- 5.a. Two is prime and three is prime.
 c. There is an integer that is even and there is an integer that is odd.
 e. Each integer is either even or odd.
 g. There is an integer that is not larger than one. [Note: that integer is one itself.]
 i. Each integer is such that if it is even then it is evenly divisible by two.
 k. Every integer is evenly divisible by one.

- m. An integer is evenly divisible by two if and only if it is even.
- o. If one is larger than some integer then it is larger than every integer.
- q. No integer is prime and evenly divisible by four.

Section 7.8E

- 1.a. $(\exists y)[Sy \ \& \ (Cy \ \& \ Ly)]$
 - c. $\sim (\forall w)[(Sw \ \& \ Lw) \supset \ Cw]$
 - e. $\sim (\forall x)[(\exists y)(Sy \ \& \ Sxy) \supset \ Sx]$
 - g. $\sim (\forall x)[(\exists y)(Sy \ \& \ (Dxy \ \vee \ Sxy)) \supset \ Sx]$
 - i. $(\forall z)[(Sz \ \& \ (\exists w)(Swz \ \vee \ Dwz)) \supset \ Lz]$
 - k. $Sr \ \vee \ (\exists y)(Sy \ \& \ Dry)$
 - m. $(Sr \ \& \ (\forall z)[(Dzr \ \vee \ Szr) \supset \ Sz]) \ \vee \ (Sj \ \& \ (\forall z)[(Dzj \ \vee \ Szj) \supset \ Sz])$
- 2.a. $(\forall x)[Ax \supset \ (\exists y)(Fy \ \& \ Exy)] \ \& \ (\forall x)[Fx \supset \ (\exists y)(Ay \ \& \ Exy)]$
 - c. $\sim (\exists y)(Fy \ \& \ Eyp)$
 - e. $\sim (\exists y)(Fy \ \& \ Eyp) \ \& \ (\exists y)(Cy \ \& \ Eyp)$
 - g. $\sim (\exists w)(Aw \ \& \ Uw) \ \& \ (\exists w)(Aw \ \& \ Fw)$
 - i. $(\exists w)[(Aw \ \& \ \sim Fw) \ \& \ (\forall y)[(Fy \ \& \ Ay) \supset \ Ewy]]$
 - k. $(\exists z)[Fz \ \& \ (\forall y)(Ay \supset \ Dzy)] \ \& \ (\exists z)[Az \ \& \ (\forall y)(Fy \supset \ Dzy)]$
 - m. $(\forall x)[(\forall y)Dxy \supset \ (Px \ \vee \ (Ax \ \vee \ Ox))]$
- 3.a. $(\forall x)[Px \supset \ (\exists y)(Syx \ \& \ Bxy)]$
 - c. $(\forall y)[(Py \ \& \ (\forall z)Bzy) \supset \ (\forall w)(Swy \supset \ Byw)]$
 - e. $(\forall w)(\forall x)[(Pw \ \& \ Sxw) \supset \ Bwx] \supset \ (\forall z)(Pz \supset \ Wz)$
 - g. $(\forall x)(\forall y)[((Px \ \& \ Syx) \ \& \ Bxy) \supset \ (\sim Nxy \ \& \ \sim Lyx)]$
 - i. $(\exists y)[Py \ \& \ (\forall z)(Pz \supset \ Byz)]$
 - k. $(\forall z)((Pz \ \& \ Uz) \supset \ [(\forall w)(Swz \supset \ Bzw) \ \vee \ (\forall w)(Swz \supset \ Gzw)])$
 - m. $(\forall w)(\forall x)[((Pw \ \& \ Sxw) \ \& \ (Bwx \ \& \ Bxw)) \supset \ (Ww \ \& \ Wx)]$
 - o. $(\exists x)(\exists y)[(Px \ \& \ Syx) \ \& \ \sim Axy]$
 - q. $(\forall y)(\forall z)[((Py \ \& \ Szy) \ \& \ \sim Lzy) \supset \ (\sim Nzy \ \& \ Bzy)]$
- 4.a. Hildegard sometimes loves Manfred.
 - c. Manfred sometimes loves Hildegard and Manfred always loves Siegfried.
 - e. If Manfred ever loves himself, then he does so whenever Hildegard loves him.
 - g. There is someone no one ever loves.
 - i. There is a time at which someone loves everyone.
 - k. There is always someone who loves everyone.
 - m. No one loves anyone all the time.
 - o. Everyone loves, at some time, himself or herself.
- 5.a. An even integer times any integer is even.
 - c. If the sum of a pair of integers is even, then either both integers are even or both are odd.
 - e. There is no prime that is larger than every prime.

- g. There are no primes such that their product is prime.
- i. There is a prime such that it times any prime is even.
- k. The product of a pair of integers is odd if and only if both members of the pair are odd.
- m. If a pair of integers are both odd, then their product is odd and their sum is even.
- o. The sum of an odd integer and an even integer is odd, and their product is even.
- q. There is an integer that is larger than one, that three is larger than, and that is prime and even.

Section 7.9E

- 1.a. $(\forall x)[(Wx \ \& \ \sim x = d) \supset Sx]$
- c. $(\forall x)[(Wx \ \& \ \sim x = d) \supset [Sx \vee (\exists y)[Sy \ \& \ (Dxy \vee Sxy)]]]$
- e. $[Sdj \ \& \ (\forall x)(Sxj \supset x = d)] \ \& \ \sim (\exists x)Dxj$
- g. $(\exists x)[(Sxr \ \& \ Sxj) \ \& \ (\forall y)[(Syr \vee Syj) \supset y = x]]$
- i. $(\exists x)(\exists y)[((Dxr \ \& \ Dyr) \ \& \ (Sx \ \& \ Sy)) \ \& \ \sim x = y]$
- k. $(\exists x)[(Sxj \ \& \ Sx) \ \& \ (\forall y)(Syj \supset y = x)] \ \& \ (\exists x)(\exists y)(([Sx \ \& \ Sy] \ \& \ (Dxj \ \& \ Dyj)) \ \& \ \sim x = y) \ \& \ (\forall z)[Dzj \supset (z = x \vee z = y)]]$

2.a. Every positive integer is less than some positive integer [or] There is no largest positive integer.

- c. There is positive integer than which no integer is less.
- e. 2 is even and prime, and it is the only positive integer that is both even and prime.
- g. The product of any pair of odd positive integers is itself odd.
- i. If either of a pair of positive integers is even, their product is even.
- k. There is exactly one prime that is greater than 5 and less than 9.

- | | |
|---|--|
| 3.a. $(\forall x)(\forall y)(Nxy \supset Nyx)$ | Symmetric only |
| c. | Neither reflexive, nor symmetric, nor transitive |
| e. $(\forall x)(\forall y)(Rxy \supset Ryx)$
$(\forall x)(\forall y)(\forall z)[(Rxy \ \& \ Ryz) \supset Rxz]$ | Symmetric and transitive |
| g. $(\forall x)Txx$
$(\forall x)(\forall y)(\forall z)[(Txy \ \& \ Tyz) \supset Txz]$ | Transitive and reflexive
(in UD: Physical objects) |
| i. $(\forall x)(\forall y)(Exy \supset Eyx)$
$(\forall x)Exx$ | Symmetric and reflexive
(in UD: People) |
| k. $(\forall x)Wxx$
$(\forall x)(\forall y)(Wxy \supset Wyx)$
$(\forall x)(\forall y)(\forall z)[(Wxy \ \& \ Wyz) \supset Wxz]$ | Symmetric, transitive, and reflexive (in UD: Physical objects) |
| m. $(\forall x)(\forall y)(\forall z)[(Axy \ \& \ Ayz) \supset Axz]$ | Transitive only |
| o. $(\forall x)Lxx$
$(\forall x)(\forall y)(Lxy \supset Lyx)$
$(\forall x)(\forall y)(\forall z)[(Lxy \ \& \ Lyz) \supset Lxz]$ | Symmetric, transitive, and reflexive (in UD: People) |

4.a. Sjc

- c. $Sjc \ \& \ (\forall x)[(Sxc \ \& \ \sim x = j) \supset Ojx]$
- e. $(\exists x)[(Dxd \ \& \ (\forall y)[(Dyd \ \& \ \sim y = x) \supset Oxy]) \ \& \ Px]$
- g. $Dcd \ \& \ (\forall x)[(Dxd \ \& \ \sim x = c) \supset Ocx]$
- i. $(\exists x)[(Sxh \ \& \ (\forall y)[(Syh \ \& \ \sim y = x) \supset Txy]) \ \& \ Mcx]$
- k. $(\exists x)[(Bx \ \& \ (\forall y)(By \supset y = x)) \ \& \ (\exists w)((Mx \ \& \ (\forall z)(Mz \supset z = w)) \ \& \ x = w)]$
- m. $(\exists x)[(Mxc \ \& \ Bxj) \ \& \ (\forall w)(Bwj \supset x = w)]$

5.a. $\sim (\exists y)a = f(y)$

- c. $(\exists x)(Px \ \& \ Ex)$
- e. $(\forall x)(\exists y)y = f(x)$
- g. $(\forall y)(Oy \supset Ef(y))$
- i. $(\forall x)(\forall y)[Ot(x,y) \supset Et(f(x), f(y))]$
- k. $(\forall x)(\forall y)[Os(x,y) \supset [(Ox \ \& \ Ey) \vee (Oy \ \& \ Ex)]]$
- m. $(\forall x)(\forall y)[(Px \ \& \ Py) \supset \sim Pt(x,y)]$
- o. $(\forall z)[(Ez \supset Eq(z)) \ \& \ (Oz \supset Oq(z))]$
- q. $(\forall x)[Ox \supset Ef(q(x))]$
- s. $(\forall x)[(Px \ \& \ \sim x = b) \supset Os(b,x)]$
- u. $(\exists x)(\exists y)[(Px \ \& \ Py) \ \& \ t(x,y) = f(s(x,y))]$

