

## CHAPTER SEVEN

### Section 7.2E

- 1.a. 'The President' is a singular term, 'Democrat' is not  
x is a Democrat  
( 'w' or 'y' or 'z' may be used in place of 'x' )
- c. 'Sarah' and 'Smith College' are the singular terms  
x attends Smith College  
Sarah attends x  
x attends y
- e. The singular terms are 'Charles' and 'Rita'  
w and Rita are brother and sister  
Charles and w are brother and sister  
w and z are brother and sister
- g. The singular terms are '2', '4', and '8'  
x times 4 is 8  
2 times x is 8  
2 times 4 is y  
x times y is 8  
x times 4 is y  
2 times x is y  
x times y is z
- i. The singular terms are '0', '0', and '0'  
z plus 0 is 0  
0 plus z is 0  
0 plus 0 is z  
w plus y is 0  
w plus 0 is y  
0 plus w is y  
w plus y is z
2. Herman is larger than Herman.  
Herman is larger than Juan.  
Herman is larger than Antonio.  
Juan is larger than Herman.  
Juan is larger than Juan.  
Juan is larger than Antonio.  
Antonio is larger than Herman.  
Antonio is larger than Juan.  
Antonio is larger than Antonio.

Herman is to the right of Herman.  
Herman is to the right of Juan.  
Herman is to the right of Antonio.  
Juan is to the right of Herman.  
Juan is to the right of Juan.  
Juan is to the right of Antonio.  
Antonio is to the right of Herman.  
Antonio is to the right of Juan.  
Antonio is to the right of Antonio.

Herman is larger than Herman but smaller than Herman.  
Herman is larger than Herman but smaller than Juan.  
Herman is larger than Herman but smaller than Antonio.  
Herman is larger than Juan but smaller than Herman.  
Herman is larger than Juan but smaller than Juan.  
Herman is larger than Juan but smaller than Antonio.  
Herman is larger than Antonio but smaller than Herman.  
Herman is larger than Antonio but smaller than Juan.  
Herman is larger than Antonio but smaller than Antonio.

Juan is larger than Herman but smaller than Herman.  
Juan is larger than Herman but smaller than Juan.  
Juan is larger than Herman but smaller than Antonio.  
Juan is larger than Juan but smaller than Herman.  
Juan is larger than Juan but smaller than Juan.  
Juan is larger than Juan but smaller than Antonio.  
Juan is larger than Antonio but smaller than Herman.  
Juan is larger than Antonio but smaller than Juan.  
Juan is larger than Antonio but smaller than Antonio.

Antonio is larger than Herman but smaller than Herman.  
Antonio is larger than Herman but smaller than Juan.  
Antonio is larger than Herman but smaller than Antonio.  
Antonio is larger than Juan but smaller than Herman.  
Antonio is larger than Juan but smaller than Juan.  
Antonio is larger than Juan but smaller than Antonio.  
Antonio is larger than Antonio but smaller than Herman.  
Antonio is larger than Antonio but smaller than Juan.  
Antonio is larger than Antonio but smaller than Antonio.

EXERCISES 7.3E

1. The *PL* analogs of the sentences of English, in the same order given in the *Solution Manual* answers to exercise 7.2E 2, are

Lhh  
Lhj  
Lha  
Ljh  
Ljj  
Lja  
Lah  
Laj  
Laa

Rhh  
Rhj  
Rha  
Rjh  
Rjj  
Rja  
Rah  
Raj  
Raa

Shhh  
Shhj  
Shha  
Shjh  
Shjj  
Shja  
Shah  
Shaj  
Shaa

Sjhh  
Sjhj  
Sjha  
Sjjh  
Sjjj  
Sjja  
Sjah  
Sjaj  
Sjaa

Sahh  
Sahj  
Saha  
Sajh  
Sajj  
Saja  
Saah  
Saaj  
Saaa

2. a. Bai  
c. Bbn  
e. Beh  
g. (Aph & Ahn) & Ank  
i. Aih  $\equiv$  Aip  
k.  $([(Lap \& Lbp) \& (Lcp \& Ldp)] \& Lep) \& \sim ([(Bap \vee Bbp) \vee (Bcp \vee Bdp)] \vee Bep)$   
m. (Tda & Tdb) & (Tdc & Tde)  
o.  $\sim ([(Tab \vee Tac) \vee (Tad \vee Tae)] \vee Taa) \& [(Lab \& Lac) \& (Lad \& Lae)]$
3. a. (Ia & Ba) &  $\sim$  Ra  
c. (Bd & Rd) & Id  
e. Ib  $\supset$  (Id & Ia)  
g. Lab & Dac  
i.  $\sim (Lca \vee Dca) \& (Lcd \& Dcd)$   
k. Acb  $\equiv$  (Sbc & Rb)  
m. (Sdc & Sca)  $\supset$  Sda  
o. (Lcb & Lba)  $\supset$  (Dca & Sca)  
q. Rd &  $\sim [Ra \vee (Rb \vee Rc)]$
4. a. UD: Margaret, Todd, Charles, and Sarah  
Gx: x is good at skateboarding  
Lx: x likes skateboarding  
Hx: x wears headgear  
Kx: x wears knee pads  
Rxy: x is more reckless than y (at skateboarding)  
Sxy: x is more skillful than y (at skateboarding)  
c: Charles  
m: Margaret  
s: Sarah  
t: Todd

$(Lm \ \& \ Lt) \ \& \ \sim \ (Gm \ \vee \ Gt)$   
 $Gc \ \& \ \sim \ Lc$   
 $Gs \ \& \ Ls$   
 $[(Hm \ \& \ Ht) \ \& \ (Hc \ \& \ Hs)] \ \& \ [(Kc \ \& \ Ks) \ \& \ \sim \ (Km \ \vee \ Kt)]$   
 $[(Rsm \ \& \ Rst) \ \& \ Rsc] \ \& \ [(Scs \ \& \ Scm) \ \& \ Sct]$

*Note:* it may be tempting to use a two-place predicate to symbolize being good at skateboarding, for example, ‘Gxy’, and another two-place predicate to symbolize liking skateboarding. So too we might use two-place predicates to symbolize wearing headgear and wearing kneepads. Doing so would require including skateboarding, headgear, and knee pads in the universe of discourse. But things are now a little murky. Skateboarding is more of an activity than a thing (although activities are often the “topics of conversation” as when we say that some people like, for example, hiking, skiing, and canoeing while others don’t). And while we might include all headgear and kneepads in our universe of discourse, we do not know which ones the characters in our passage wear, so we would be hard pressed to name the favored items.

Moreover, here there is no need to invoke these two-place predicates because here we are not asked to investigate logical relations that can only be expressed with two-place predicates. The case would be different if the passage included the sentence ‘If Sarah is good at anything she is good at sailing’ and we were asked to show that it follows from the passage that Sarah is good at sailing. (On the revised scenario we are told that Sarah is good at skateboarding, and that if she is good at anything—she is, skateboarding—she is good at sailing. So she is good at sailing. Here we are treating skateboarding as *something*, something Sarah is good at. But we will leave these complexities until we have fully developed the language *PL*.)

c. One appropriate symbolization key is

UD: Andrew, Christopher, Amanda  
Hz: z is a hiker  
Mz: z is a mountain climber  
Kz: z is a kayaker  
Sz: z is a swimmer  
Lzw: z likes w  
Nzw: z is nuts about w  
a: Andrew  
c: Christopher  
m: Amanda

$(Ha \ \& \ Hc) \ \& \ \sim \ (Ma \ \vee \ Mc)$   
 $(Hm \ \& \ Mm) \ \& \ Km$   
 $(Ka \ \vee \ Kc) \ \& \ \sim \ (Ka \ \& \ Kc)$   
 $\sim \ [(Sa \ \vee \ Sc) \ \vee \ Sm]$   
 $((Lac \ \& \ Lca) \ \& \ [(Lam \ \& \ Lma) \ \& \ (Lmc \ \& \ Lcm)]) \ \& \ (Nma \ \& \ Nam)$

**Section 7.4E**

- 1.a.  $(\forall z)Bz$ 
  - c.  $\sim (\exists x)Bx$
  - e.  $(\exists x)Bx \ \& \ (\exists x)Rx$
  - g.  $(\exists z)Rz \supset (\exists z)Bz$
  - i.  $(\forall y)By \equiv \sim (\exists y)Ry$
- 2.a.  $(\exists x)Ox \ \& \ (\exists x)Ex$ 
  - c.  $\sim (\exists x)Lxa$
  - e.  $(\forall x)Gx$
  - g.  $(\exists x)(Px \ \& \ Ex)$
  - i.  $(\forall y)[(Py \ \& \ Lby) \supset Ey]$
  - k.  $(\exists y)(Lby \ \& \ Lyc)$
- 3.a.  $Pj \supset (\forall x)Px$ 
  - c.  $(\exists y)Py \supset (Pj \ \& \ Pr)$
  - e.  $\sim Pr \supset \sim (\exists x)Px$
  - g.  $(Pj \supset Pr) \ \& \ (Pr \supset (\forall x)Px)$
  - i.  $(\forall y)Sy \ \& \ \sim (\forall y)Py$
  - k.  $(\forall x)Sx \supset (\exists y)Py$

**Section 7.5E**

- 1.a. A formula but not a sentence (an open sentence): the 'z' in 'Zz' is free.
  - c. A formula and a sentence.
  - e. A formula but not a sentence (an open sentence): the 'x' in 'Fxz' is free.
  - g. A formula and a sentence.
  - i. Not a formula. ' $\sim (\exists x)$ ' is an expression of *SL*, but ' $(\sim \exists x)$ ' is not.
  - k. Not a formula. Since there is no 'y' in 'Lxx', ' $(\exists y)Lxx$ ' is not a formula. Hence, neither is ' $(\exists x)(\exists y)Lxx$ '.
  - m. A formula and a sentence.
  - o. A formula but not a sentence (an open sentence): 'w' in 'Fw' is free.
- 2.a. A sentence. The subformulas are

$(\exists x)(\forall y)Byx$	$(\exists x)$
$(\forall y)Byx$	$(\forall y)$
$Byx$	None

c. Not a sentence. The 'x' in '(Bg  $\supset$  Fx)' is free. The subformulas are

$(\forall x)(\sim Fx \ \& \ Gx) \equiv (Bg \supset Fx)$	$\equiv$
$(\forall x)(\sim Fx \ \& \ Gx)$	$(\forall x)$
$Bg \supset Fx$	$\supset$
$\sim Fx \ \& \ Gx$	$\&$
$\sim Fx$	$\sim$
$Gx$	None
$Bg$	None
$Fx$	None

e. Sentence. The subformulas are

$\sim (\exists x)Px \ \& \ Rab$	$\&$
$\sim (\exists x)Px$	$\sim$
$Rab$	None
$(\exists x)Px$	$(\exists x)$
$Px$	None

g. Sentence. The subformulas are

$\sim [\sim (\forall x)Fx \equiv (\exists w) \sim Gw] \supset Maa$	$\supset$
$\sim [\sim (\forall x)Fx \equiv (\exists w) \sim Gw]$	$\sim$
$Maa$	None
$\sim (\forall x)Fx \equiv (\exists w) \sim Gw$	$\equiv$
$\sim (\forall x)Fx$	$\sim$
$(\exists w) \sim Gw$	$(\exists w)$
$(\forall x)Fx$	$(\forall x)$
$Fx$	None
$\sim Gw$	$\sim$
$Gw$	None

i. Sentence. The subformulas are

$\sim \sim \sim (\exists x)(\forall z)(Gxaz \vee \sim Hazb)$	$\sim$
$\sim \sim (\exists x)(\forall z)(Gxaz \vee \sim Hazb)$	$\sim$
$\sim (\exists x)(\forall z)(Gxaz \vee \sim Hazb)$	$\sim$
$(\exists x)(\forall z)(Gxaz \vee \sim Hazb)$	$(\exists x)$
$(\forall z)(Gxaz \vee \sim Hazb)$	$(\forall z)$
$Gxaz \vee \sim Hazb$	$\vee$
$Gxaz$	None
$\sim Hazb$	$\sim$
$Hazb$	None

k. Sentence. The subformulas are

$(\exists x)[Fx \supset (\forall w)(\sim Gx \supset \sim Hwx)]$	$(\exists x)$
$Fx \supset (\forall w)(\sim Gx \supset \sim Hwx)$	$\supset$
$Fx$	None
$(\forall w)(\sim Gx \supset \sim Hwx)$	$(\forall w)$
$\sim Gx \supset \sim Hwx$	$\supset$
$\sim Gx$	$\sim$
$\sim Hwx$	$\sim$
$Gx$	None
$Hwx$	None

m. A sentence. The subformulas are

$(Hb \vee Fa) \equiv (\exists z)(\sim Fz \ \& \ Gza)$	$\equiv$
$Hb \vee Fa$	$\vee$
$(\exists z)(\sim Fz \ \& \ Gza)$	$(\exists z)$
$Hb$	None
$Fa$	None
$\sim Fz \ \& \ Gza$	$\&$
$\sim Fz$	$\sim$
$Gza$	None
$Fz$	None

3.a. $(\forall x)(Fx \supset Ga)$	Quantified
c. $\sim (\forall x)(Fx \supset Ga)$	Truth-functional
e. $\sim (\exists x)Hx$	Truth-functional
g. $(\forall x)(Fx \equiv (\exists w)Gw)$	Quantified
i. $(\exists w)(Pw \supset (\forall y)(Hy \equiv \sim Kyw))$	Quantified
k. $\sim [(\exists w)(Jw \vee Nw) \vee (\exists w)(Mw \vee Lw)]$	Truth-functional
m. $(\forall z)Gza \supset (\exists z)Fz$	Truth-functional
o. $(\exists z) \sim Hza$	Quantified
q. $(\forall x) \sim Fx \equiv (\forall z) \sim Hza$	Truth-functional

4.a.  $Maa \ \& \ Fa$

c. $\sim (Ca \equiv \sim Ca)$
e. $(Fa \ \& \ \sim Gb) \supset (Bab \vee Bba)$
g. $\sim (\exists z)Naz \equiv (\forall w)(Mww \ \& \ Naw)$
i. $Fab \equiv Gba$
k. $\sim (\exists y)(Hay \ \& \ Hya)$
m. $(\forall y)[(Hay \ \& \ Hya) \supset (\exists z)Gza]$

5.a. $(\forall y)Ray \supset Byy$	No
c. $(\forall y)(Rwy \supset Byy)$	No
e. $(\forall y)(Ryy \supset Byy)$	No
g. $(Ray \supset Byy)$	No
i. $Rab \supset Bbb$	No
6.a. $(\forall y) \sim Ray \equiv Paa$	Yes
c. $(\forall y) \sim Ray \equiv Pba$	No
e. $(\forall y)(\sim Ryy \equiv Paa)$	No
g. $(\forall y) \sim Raw \equiv Paa$	No

### Section 7.6E

1.a. A-sentence	$(\forall y)(Py \supset Cy)$
c. O-sentence	$(\exists w)(Dw \ \& \ \sim Sw)$
e. I-sentence	$(\exists z)(Nz \ \& \ Bz)$
g. E-sentence	$(\forall x)(Px \supset \sim Sx)$
i. A-sentence	$(\forall w)(Pw \supset Mw)$
k. A-sentence	$(\forall y)(Sy \supset Cy)$
m. E-sentence	$(\forall y)(Ky \supset \sim Sy)$
o. E-sentence	$(\forall y)(Qy \supset \sim Zy)$
2.a. $(\forall y)(By \supset Ly)$	
c. $(\forall z)(Rz \supset \sim Lz)$	
e. $(\exists x)Bx \ \& \ (\exists x)Rx$	
g. $[(\exists z)Bz \ \& \ (\exists z)Rz] \ \& \ \sim (\exists z)(Bz \ \& \ Rz)$	
i. $(\exists y)By \ \& \ [(\exists y)Sy \ \& \ (\exists y)Ly]$	
k. $(\forall w)(Cw \supset Rw) \ \& \ \sim (\forall w)(Rw \supset Cw)$	
m. $(\forall y)Ry \vee [(\forall y)By \vee (\forall y)Gy]$	
o. $(\exists w)(Rw \ \& \ Sw) \ \& \ (\exists w)(Rw \ \& \ \sim Sw)$	
q. $(\exists x)Ox \ \& \ (\forall y)(Ly \supset \sim Oy)$	

3.a. An I-sentence and the corresponding O-sentence of *PL* can both be true. Consider the English sentences ‘Some positive integers are even’ and ‘Some positive integers are not even’. Where the UD is positive integers and ‘Ex’ is interpreted as ‘x is even’, these can be symbolized as ‘ $(\exists x)Ex$ ’ and ‘ $(\exists x) \sim Ex$ ’, respectively, and both sentences of *PL* are true.

An I-sentence and an O-sentence can also both be false. Consider ‘Some tiggers are fast’ and ‘Some tiggers are not fast’. Where the UD is mammals, ‘Tx’ is interpreted as ‘x is a tigger’ and ‘Fx’ as ‘x is fast’, these become, respectively, ‘ $(\exists x)(Tx \ \& \ Fx)$ ’ and ‘ $(\exists x)(Tx \ \& \ \sim Fx)$ ’. As there are no tiggers, both sentences of *PL* are false. Note, however, that there cannot be an I-sentence and a corresponding O-sentence of the sorts  $(\exists x)A$  and  $(\exists x) \sim A$ , where **A** is an atomic formula and both the I-sentence and the O-sentence are false. For however **A** is interpreted, either there is something that satisfies it, or there is not. In the first instance  $(\exists x)A$  is true, in the second  $(\exists x) \sim A$  is true.

**Section 7.7E**

- 1.a.  $(\forall z)(Pz \supset Hz)$   
 c.  $(\exists z)(Pz \ \& \ Hz)$   
 e.  $(\forall w)[(Hw \ \& \ Pw) \supset \sim Iw]$   
 g.  $\sim (\forall x)[(Px \vee Ix) \supset Hx]$   
 i.  $(\forall y)[(Iy \ \& \ Hy) \supset Ry]$   
 k.  $(\exists z)Iz \supset Ih$   
 m.  $(\exists w)Iw \supset (\forall x)(Rx \supset Ix)$   
 o.  $\sim (\exists y)[Hy \ \& \ (Py \ \& \ Iy)]$   
 q.  $(\forall z)(Pz \supset Iz) \supset \sim (\exists z)(Pz \ \& \ Hz)$   
 s.  $(\forall w)(Rw \supset [(Lw \ \& \ Iw) \ \& \ \sim Hw])$
- 2.a.  $(\forall w)(Lw \supset Aw)$   
 c.  $(\forall x)(Lx \supset Fx) \ \& \ (\forall x)(Tx \supset \sim Fx)$   
 e.  $(\exists y)[(Fy \ \& \ Ly) \ \& \ Cdy]$   
 g.  $(\forall z)[(Lz \vee Tz) \supset Fz]$   
 i.  $(\exists w)(Tw \ \& \ Fw) \ \& \ \sim (\forall w)(Tw \supset Fw)$   
 k.  $(\forall x)[(Lx \ \& \ Cbx) \supset (Ax \ \& \ \sim Fx)]$   
 m.  $(\exists z)(Lz \ \& \ Fz) \supset (\forall w)(Tw \supset Fw)$   
 o.  $\sim Fb \ \& \ Bb$
- 3.a.  $(\forall x)(Ex \supset Yx)$   
 c.  $(\exists y)(Ey \ \& \ Yy) \ \& \ \sim (\forall y)(Ey \supset Yy)$   
 e.  $(\exists z)(Ez \ \& \ Yz) \supset (\forall x)(Lx \supset Yx)$   
 g.  $(\forall w)[(Ew \ \& \ Sw) \supset Yw]$   
 i.  $(\forall w)[(Lw \ \& \ Ew) \supset (Yw \ \& \ Iw)]$   
 k.  $(\forall x)[(Ex \vee Lx) \supset (Yx \supset Ix)]$   
 m.  $\sim (\exists z)[(Pz \ \& \ \sim Iz) \ \& \ Yz]$   
 o.  $(\forall x)[(Ex \ \& \ Rxx) \supset Yx]$   
 q.  $(\forall x)[(Ex \vee Lx) \ \& \ (Rx \vee Yx)] \supset Rxx$   
 s.  $(\forall z)[(Yz \ \& \ (Lz \ \& \ Ez))] \supset Rzz$
- 4.a.  $(\forall x)[Px \supset (Ux \ \& \ Ox)]$   
 c.  $(\forall z)[Az \supset \sim (Oz \vee Uz)]$   
 e.  $(\forall w)(Ow \equiv Uw)$   
 g.  $(\exists y)(Py \ \& \ Uy) \ \& \ (\forall y)[(Py \ \& \ Ay) \supset \sim Uy]$   
 i.  $(\exists z)[Pz \ \& \ (Oz \ \& \ Uz)] \ \& \ (\forall x)[Sx \supset (Ox \ \& \ Ux)]$   
 k.  $((\exists x)(Sx \ \& \ Ux) \ \& \ (\exists x)(Px \ \& \ Ux)) \ \& \ \sim (\exists x)(Ax \ \& \ Ux)$
- 5.a. Two is prime and three is prime.  
 c. There is an integer that is even and there is an integer that is odd.  
 e. Each integer is either even or odd.  
 g. There is an integer that is not larger than one. [Note: that integer is one itself.]  
 i. Each integer is such that if it is even then it is evenly divisible by two.  
 k. Every integer is evenly divisible by one.

- m. An integer is evenly divisible by two if and only if it is even.
- o. If one is larger than some integer then it is larger than every integer.
- q. No integer is prime and evenly divisible by four.

### Section 7.8E

- 1.a.  $(\exists y)[Sy \ \& \ (Cy \ \& \ Ly)]$ 
  - c.  $\sim (\forall w)[(Sw \ \& \ Lw) \supset Cw]$
  - e.  $\sim (\forall x)[(\exists y)(Sy \ \& \ Sxy) \supset Sx]$
  - g.  $\sim (\forall x)[(\exists y)(Sy \ \& \ (Dxy \ \vee \ Sxy)) \supset Sx]$
  - i.  $(\forall z)[(Sz \ \& \ (\exists w)(Swz \ \vee \ Dwz)) \supset Lz]$
  - k.  $Sr \ \vee \ (\exists y)(Sy \ \& \ Dry)$
  - m.  $(Sr \ \& \ (\forall z)[(Dzr \ \vee \ Szr) \supset Sz]) \ \vee \ (Sj \ \& \ (\forall z)[(Dzj \ \vee \ Szj) \supset Sz])$
- 2.a.  $(\forall x)[Ax \supset (\exists y)(Fy \ \& \ Exy)] \ \& \ (\forall x)[Fx \supset (\exists y)(Ay \ \& \ Exy)]$ 
  - c.  $\sim (\exists y)(Fy \ \& \ Eyp)$
  - e.  $\sim (\exists y)(Fy \ \& \ Eyp) \ \& \ (\exists y)(Cy \ \& \ Eyp)$
  - g.  $\sim (\exists w)(Aw \ \& \ Uw) \ \& \ (\exists w)(Aw \ \& \ Fw)$
  - i.  $(\exists w)[(Aw \ \& \ \sim Fw) \ \& \ (\forall y)[(Fy \ \& \ Ay) \supset Ewy]]$
  - k.  $(\exists z)[Fz \ \& \ (\forall y)(Ay \supset Dzy)] \ \& \ (\exists z)[Az \ \& \ (\forall y)(Fy \supset Dzy)]$
  - m.  $(\forall x)[(\forall y)Dxy \supset (Px \ \vee \ (Ax \ \vee \ Ox))]$
- 3.a.  $(\forall x)[Px \supset (\exists y)(Syx \ \& \ Bxy)]$ 
  - c.  $(\forall y)[(Py \ \& \ (\forall z)Bzy) \supset (\forall w)(Swy \supset Byw)]$
  - e.  $(\forall w)(\forall x)[(Pw \ \& \ Sxw) \supset Bwx] \supset (\forall z)(Pz \supset Wz)$
  - g.  $(\forall x)(\forall y)[((Px \ \& \ Syx) \ \& \ Bxy) \supset (\sim Nxy \ \& \ \sim Lyx)]$
  - i.  $(\exists y)[Py \ \& \ (\forall z)(Pz \supset Byz)]$
  - k.  $(\forall z)((Pz \ \& \ Uz) \supset [(\forall w)(Swz \supset Bzw) \ \vee \ (\forall w)(Swz \supset Gzw)])$
  - m.  $(\forall w)(\forall x)[((Pw \ \& \ Sxw) \ \& \ (Bwx \ \& \ Bxw)) \supset (Ww \ \& \ Wx)]$
  - o.  $(\exists x)(\exists y)[(Px \ \& \ Syx) \ \& \ \sim Axy]$
  - q.  $(\forall y)(\forall z)[((Py \ \& \ Szy) \ \& \ \sim Lzy) \supset (\sim Nzy \ \& \ Bzy)]$
- 4.a. Hildegard sometimes loves Manfred.
  - c. Manfred sometimes loves Hildegard and Manfred always loves Siegfried.
  - e. If Manfred ever loves himself, then he does so whenever Hildegard loves him.
  - g. There is someone no one ever loves.
  - i. There is a time at which someone loves everyone.
  - k. There is always someone who loves everyone.
  - m. No one loves anyone all the time.
  - o. Everyone loves, at some time, himself or herself.
- 5.a. An even integer times any integer is even.
  - c. If the sum of a pair of integers is even, then either both integers are even or both are odd.
  - e. There is no prime that is larger than every prime.



4.a. Sjc

- c.  $Sjc \ \& \ (\forall x)[(Sxc \ \& \ \sim x = j) \supset Ojx]$
- e.  $(\exists x)[(Dxd \ \& \ (\forall y)[(Dyd \ \& \ \sim y = x) \supset Oxy]) \ \& \ Px]$
- g.  $Dcd \ \& \ (\forall x)[(Dxd \ \& \ \sim x = c) \supset Ocx]$
- i.  $(\exists x)[(Sxh \ \& \ (\forall y)[(Syh \ \& \ \sim y = x) \supset Txy]) \ \& \ Mcx]$
- k.  $(\exists x)[(Bx \ \& \ (\forall y)(By \supset y = x)) \ \& \ (\exists w)((Mx \ \& \ (\forall z)(Mz \supset z = w)) \ \& \ x = w)]$
- m.  $(\exists x)[(Mxc \ \& \ Bxj) \ \& \ (\forall w)(Bwj \supset x = w)]$

5.a.  $\sim (\exists y)a = f(y)$

- c.  $(\exists x)(Px \ \& \ Ex)$
- e.  $(\forall x)(\exists y)y = f(x)$
- g.  $(\forall y)(Oy \supset Ef(y))$
- i.  $(\forall x)(\forall y)[Ot(x,y) \supset Et(f(x), f(y))]$
- k.  $(\forall x)(\forall y)[Os(x,y) \supset [(Ox \ \& \ Ey) \vee (Oy \ \& \ Ex)]]$
- m.  $(\forall x)(\forall y)[(Px \ \& \ Py) \supset \sim Pt(x,y)]$
- o.  $(\forall z)[(Ez \supset Eq(z)) \ \& \ (Oz \supset Oq(z))]$
- q.  $(\forall x)[Ox \supset Ef(q(x))]$
- s.  $(\forall x)[(Px \ \& \ \sim x = b) \supset Os(b,x)]$
- u.  $(\exists x)(\exists y)[(Px \ \& \ Py) \ \& \ t(x,y) = f(s(x,y))]$

