Philosophy 220

Truth Functional Properties Expressed in terms of Consistency

The concepts of truth-functional logic:

* Truth-functional:
  * Truth
  * Falsity
  * Indeterminacy
  * Entailment
  * Validity
  * Equivalence
  * Consistency

The concepts of truth-functional logic:

- The section of the text from 110-113 aims to demonstrate that all of the other concepts of truth-functional logic can be explained in terms of truth-functional consistency.
- As it happens, all of the concepts of truth-functional logic can be explained in terms of any of the other concepts of truth-functional logic listed previously.

Why Consistency?

- If all of the other concepts of truth-functional logic can be explained via truth functional consistency, then a system that determines consistency can determine all of the other concepts as well.
- We will be replacing truth-tables with a system based on consistency (but that is much easier to learn if you already are very familiar with truth-tables).
- This new system, called the ‘semantic tree system’ will be our primary system for determining validity, entailment, equivalency, etc. for the remainder of the course.

Truth-Functional Consistency (Review)

- A set of sentences of SL is truth-functionally consistent if and only if there is at least one truth value assignment [of the constituents of the set of sentences] on which all the members of the set are true.

Truth-Functional Falsity

Definition

* A sentence of SL is truth-functionally false if and only if it is false on every possible truth-value assignment of its constituents.

Explained via consistency

* A sentence $\mathbb{P}$ is truth-functionally false if and only if {$\mathbb{P}$} is truth-functionally inconsistent.
* Since inconsistent sets are sets that can never all be true at the same time, and since the unit set of $\mathbb{P}$ has only one member, it must always be false to be inconsistent.

Truth-Functional Truth

Definition

* A sentence of SL is truth-functionally true if and only if it is true on every possible truth-value assignment of its constituents.

Explained via Consistency

* A sentence **P** is truth-functionally true if and only if {~**P**} is truth-functionally inconsistent.
* Since inconsistent sets are sets that can never be true at the same time, and since the unit set ~**P** has only one member and is a negation.

Truth-Functional Indeterminacy

Definition

* A sentence of SL is truth-functionally indeterminate if and only if it is neither truth-functionally true nor truth-functionally false.

Explained via consistency

* A sentence ℙ is truth-functionally indeterminate if and only if both {~ℙ} and {ℙ} are truth-functionally consistent.
* Since ℙ is truth functionally true or false if one of the above sets is inconsistent

Truth-Functional Equivalence

Definition

* Sentences ℙ and ℚ of SL are truth-functionally equivalent if and only if there is no truth value assignment [for the components of ℙ and ℚ] on which ℙ and ℚ have different truth-values.

Explained via consistency

* Sentences ℙ and ℚ of SL are truth-functionally equivalent if and only if {~(ℙ ≡ ℚ)} is truth-functionally inconsistent
* Since only truth-functionally false sentences are inconsistent as sole members of a set, the negation of a sentence asserting that ℙ and ℚ have different truth-values being truth functionally false means that ℙ and ℚ must have the same truth-value.

A new symbol:

* To define validity and entailment by means of consistency, it is useful to introduce a new symbol:
* '∪' is the union symbol.
* The union symbol is used to express the combination of two sets together or to express the combination of a set and a sentence.
* Example: {A, B, C} ∪ D is {A, B, C, D}

Truth-functional entailment

Definition

- A set Γ of sentences of SL truth-functionally entails a sentence $\mathbb{P}$ if and only if there is no truth-value assignment on which every member of Γ is true and $\mathbb{P}$ is false.

Explained via Consistency

- $\Gamma \models \mathbb{P}$ if and only if $\Gamma \cup \{\sim\mathbb{P}\}$ is truth-functionally inconsistent.
- Next slide contains more detailed rationale…

$\Gamma \models \mathbb{P}$ if and only if $\Gamma \cup \{\sim\mathbb{P}\}$ is truth-functionally inconsistent.

- If the set Γ entails $\mathbb{P}$, then there is no truth-value assignment that makes the members of Γ true while $\mathbb{P}$ is false. That means that whenever the members of Γ are all true, $\mathbb{P}$ is also, so $\Gamma \cup \{\sim\mathbb{P}\}$ would be inconsistent.
- Side note: If Γ is inconsistent to begin with, then $\Gamma \cup \{\sim\mathbb{P}\}$ is still inconsistent, and Γ still entails $\mathbb{P}$, because inconsistent sets entail anyhting.

Truth-Functional Validity

- Since validity is simply a special case of entailment, the same procedure can demonstrate that validity can be described in terms of consistency.
- If an argument is valid, then the union of the set of its premises and the negation of its conclusion will form a truth-functionally inconsistent set.