Philosophy 220

Truth-Functional Entailment and Validity

- A set Γ of sentences of SL truth-functionally entails a sentence **P** if and only if there is no truth-value assignment on which every member of Γ is true and **P** is false.
- We use the double-turnstile, '⊨' to indicate entailment, while we use the negated turnstile, '⊭' to indicate non-entailment.
- Also, notice the use of 'Γ' as a metavariable ranging over sets of sentences of SL.

Truth-Functional Entailment

Checking for Entailment

- On a full truth-table, An entailment relation holds between some set Γ of sentences of SL and $\mathbb{P}$ if and only if there is no row of the truth table in which every member of Γ is true while $\mathbb{P}$ is false.
- A partial truth table can prove non-entailment by arriving at a coherent truth-value assignment while assuming every member of Γ is true while $\mathbb{P}$ is false.
- If no such coherent truth-value assignment exists, then the entailment relation holds.

- To the left of the entailment symbol is always either a set {…} or a metavariable ranging over sets of sentences of SL.
- To the right of the entailment symbol is always either a sentence of SL or a metavariable ranging over sentences of SL.
- When nothing is to the left of the entailment symbol (as in '$\models \mathbb{Q}$') it is to be understood that this is shorthand for saying that $\mathbb{Q}$ is entailed by the empty set, symbolized '∅', which is a set that contains no members.

Other Notation Issues

Self Test 1

- If $\{\mathbb{P}\} \models \mathbb{Q}$ and $\{\mathbb{Q}\} \models \mathbb{P}$, does this mean that $\mathbb{P}$ and $\mathbb{Q}$ are truth-functionally equivalent?

- If $\{\mathbb{P}\} \models \mathbb{Q}$ and $\{\mathbb{Q}\} \models \mathbb{P}$, does this mean that $\mathbb{P}$ and $\mathbb{Q}$ are truth-functionally equivalent?
- YES.
- If there are no conditions under which $\mathbb{P}$ is true while $\mathbb{Q}$ is false, and also no conditions under which $\mathbb{Q}$ is true while $\mathbb{P}$ is false, then $\mathbb{P}$ and $\mathbb{Q}$ always have a truth-value in common, and so are truth-functionally equivalent.

Self Test 1

- If $\varnothing \models \mathbb{Q}$, what do we know for sure about $\mathbb{Q}$?

Self Test 2

- If $\varnothing \vDash \mathbb{Q}$, what do we know for sure about $\mathbb{Q}$?
- We know that $\mathbb{Q}$ is truth-functionally true, because only truth-functionally true sentences are true even when nothing else is.

Self Test 2

- What does Γ truth-functionally entail if it is truth-functionally inconsistent?

Self Test 3

- What does Γ truth-functionally entail if it is truth-functionally inconsistent?
- Any sentence of SL is truth-functionally entailed by any truth-functionally inconsistent set.
- This is because there will never be a case in which all of the sentences in Γ are true, so it will never be the case that all of the members of Γ are true while ℙ is false.
- This could be called 'trivial entailment'.

Self Test 3

- Arguments occur when some sentence or sentences are designated as premises while another sentence is designated as the conclusion.
- Validity is a special case of entailment that applies to arguments.
- An argument is truth functionally valid if and only if its conclusion is truth functionally entailed by the set of sentences comprised by its premises.

Truth-Functional Validity

- If P, Q, and R are each premises, and S is the conclusion of a truth-functionally valid argument, then the following truth-functional entailment relation must hold:
  - {P, Q, R} ⊨ S
- Also, the following material conditional must be true:
  - (P & (Q & R))⊃ S

Validity, Entailment, and the Material Conditional