Philosophy 220

Tautology, Contradiction, and Contingency

Review

- A sentence in natural language is **logically true** if and only if it cannot (logically) be false. (Tautology)
- A sentence in natural language is **logically false** if and only if cannot (logically) be true. (Contradiction)
- A sentence in natural language is **logically indeterminate** if and only if it is neither logically true nor logically false (Contingent).

In SL:

- In SL, the concepts of logical truth, logical falsity, and logical indeterminacy are given (by the text authors) distinct names to indicate that they apply to sentences of SL as opposed to sentences of natural language.
- We shall not follow them in this.

Tautology

- A sentence of SL (or anything else) is a tautology if and only if it is true on every possible truth-value assignment of its constituents.

Contradiction

- A sentence of SL (or anything else) is a contradiction if and only if it is false on every possible truth-value assignment of its constituents.

Contingent

- A sentence of SL (or anything else) is contingent if and only if it is neither a tautology nor a contradiction.

Checking for truth-functional status

- We will here introduce the use of partial truth tables to check for tautology, contradiction, and contingency.
- The idea is to see whether a counterexample to tautology or contradiction is possible, and conclude what we may from that.

Test whether '(P v Q) ⊃ (R & S)' is a contradiction

P	Q	R	S	(P v Q)	⊃	(R & S)
					T	

To test for contradiction, we will look for a counterexample. So we will assume that the main connective is true. If it is impossible to get a coherent truth value assignment for P, Q, R, and S, then we may conclude that our sentence is a contradiction. If we get a coherent truth value assignment for P, Q, R, and S, then we will have demonstrated that our sentence is not a contradiction, because it can be true.

Test whether ‘(P v Q) ⊃ (R & S)’ is a contradiction

P	Q	R	S	(P v Q)	⊃	(R & S)
				T	**T**	T

Our main connective is a conditional, and could be true under three conditions, so we’ll pick one to start with.

If we get an impossible result out of this one, we have two more to try before concluding that our sentence is a contradiction.

Test whether '(P v Q) ⊃ (R & S)' is a contradiction

P	Q	R	S	(P v Q)	⊃	(R & S)
		T	**T**	**T**	**T**	**T**

Since a true R & S means a true R and a true S, we can fill those in.

Test whether '(P v Q) ⊃ (R & S)' is a contradiction

P	Q	R	S	(P v Q)	⊃	(R & S)
T	**T**	**T**	**T**	**T**	**T**	**T**

P v Q can be true in one of three ways, all of which are compatible with everything else on the table. So we'll just pick one.

We have here shown that the above sentence can be true, therefore it cannot be a contradiction.

We must go on to test this one for tautology now.

Test whether '(P v Q) ⊃ (R & S)' is a contradiction

P	Q	R	S	(P v Q)	⊃	(R & S)
					F	

Remember that to see if it is a tautology we must see if we can generate a counterexample. If it can be false, then it is not a tautology. If it cannot be false, then it is a tautology.

Test whether ‘(P v Q) ⊃ (R & S)’ is a tautology

P	Q	R	S	(P v Q)	⊃	(R & S)
				T	F	F

If a conditional is false, its antecedent is true and its consequent is false.

Test whether '(P v Q) ⊃ (R & S)' is a tautology

P	Q	R	S	(P v Q)	⊃	(R & S)
T	**T**			**T**	**F**	**F**

P v Q will be true in three different cases, so we'll pick one to start with.

Test whether '(P v Q) ⊃ (R & S)' is a tautology

P	Q	R	S	(P v Q)	⊃	(R & S)
T	**T**	**F**	**F**	**T**	**F**	**F**

R & S will be false in three different cases, so we'll pick one to start with.

This results in a coherent truth value assignment for P, Q, R, and S the result of which is that our sentence is false.

Test whether '(P v Q) ⊃ (R & S)' is a tautology

P	Q	R	S	(P v Q)	⊃	(R & S)
T	**T**	**F**	**F**	**T**	**F**	**F**

Since our sentence is neither a tautology nor a contradiction, we can conclude that it is contingent.

Test whether '~B ⊃ [(B v D) ⊃ D]' is a tautology

B	D	~B	⊃	[(B v D)	⊃ D]

Test whether '~B ⊃ [(B v D) ⊃ D]' is a tautology

B	D	~B	⊃	[(B v D)	⊃ D]
			F		

A tautology will never be false, so if we plug in a value of F for the main connective and get a coherent truth assignment for B and D, we know that the sentence *can be* false, and so cannot be a tautology. If assuming a false sentence prevents us from arriving at ANY coherent truth value assignments for B and D, then the sentence cannot be false, and so must be a tautology.

Test whether '~B ⊃ [(B v D) ⊃ D]' is a tautology

B	D	~B	⊃	[(B v D)	⊃ D]
			F		

The main connective is a conditional, and we are assuming it is false. A conditional is false only under the conditions that its antecedent is true and its consequent false.

Test whether '~B ⊃ [(B v D) ⊃ D]' is a tautology

B	D	~B	⊃	[(B v D)	⊃ D]
		T	**F**		**F**

The main connective is a conditional, and we are assuming it is false. A conditional is false only under the conditions that its antecedent is true and its consequent false.

Test whether '~B ⊃ [(B v D) ⊃ D]' is a tautology

B	D	~B	⊃	[(B v D)	⊃ D]
F		**T**	**F**		**F**

Now we have a couple of things we must do to finish the table. First, we notice that if ~B is true, then B must be false.

Test whether '~B ⊃ [(B v D) ⊃ D]' is a tautology

B	D	~B	⊃	[(B v D)	⊃ D]
F	**F**	**T**	**F**	**T**	**F**

If (B v D) ⊃ D is false, then B v D must be true while D is false.

Test whether '~B ⊃ [(B v D) ⊃ D]' is a tautology

B	D	~B	⊃	[(B v D)	⊃ D]
F	**F**	**T**	**F**	**T**	**F**

FAIL!

If (B v D) is true, then either B or D must be true.

Since there is nothing else we could have done at any point, we have shown that it is impossible for ~B ⊃ [(B v D) ⊃ D] to be false, so we conclude that it is a tautology.

If it had been possible for ~B ⊃ [(B v D) ⊃ D] to be false, then we could have concluded that it was not a tautology.

(Note for future: We have just used Reductio ad Absurdum!)

Full Truth-Tables

- Of course, one can always simply do a full truth-table and check to see if the column for the main connective is all true (for tautologies), all false (for contradictions), or a mix of the two (for contingent sentences).