# Philosophy 220

Syntax of PL 2

# 2 States of Variables:

- Bound variable: an occurrence of a variable **x** in a formula **P** of PL that is within the scope of an x-quantifier.
- E.g. in '(∃z)Pz', 'z' is a bound variable
- Free variable: an occurrence of a variable **x** in a formula **P** of PL that is not bound.

# Sentences of PL:

- Remember: All uses of the vocabulary of PL are **expressions** of PL.
- An **expression** of PL is a **formula** of PL if and only if it can be built from the rules of syntax for **formulae** (p.299).
- A **formula** of PL is a **sentence** of PL if and only if it contains no free variable.

# Scope of Quantifiers:

- Note that finding the main logical operator of a sentence treats a quantifier like a negation because it's a unary operator.

- For example:
  - (∀x)(Px & Qa) has as its main operator '(∀x)'
  - (∀x)Px & Qa has as its main operator '&'
  - ~(Px & Qa) has as its main operator '~' (formula but not sentence)
  - ~Px & Qa has as its main operator '&' (formula but not sentence)

# Substitution Instances of Quantified Sentences:

- The notation '**P(a/x)**' indicates a formula **P** with individual constant **a** in the place of each occurrence of variable **x**.
- A substitution instance:
  - 1) may only be formed from a sentence whose main logical operator is a quantifier
  - 2) may only be formed by dropping the main logical operator
- So Pa is a substitution instance of (∀x)Px
- (∀y)Pay is **not** a substitution instance of (∀y)(∀x)Pxy (rule 2)
- ~Pa is **not** a substitution instance of ~(∀x)Px (rule 1)