PHILOSOPHY 220

SYNTAX OF PL 1

VOCABULARY OF PL

- Predicates of PL: The capital letters A-Z, with or w/o numerical subscripts.
  - An n-place predicate is indicated by the presence of exactly n primes (‘).
- Individual Terms of PL:
  - Individual constants: lowercase letters a-v, with or without numerical subscripts
  - Individual variables: lowercase letters w-z, with or without numerical subscripts
- Truth-Functional, connectives: ~ & ∨ ⊃ ≡
- Quantifiers: ∀ ∃
- Punctuation marks: ()[]

METAVARIABLES:

- As with SL, we will use the characters '**P**', '**Q**', '**R**', etc. as metavariables ranging over all expressions of PL.
- We will also use the character '**x**' to range over all individual variables of PL.
- We will use the character '**a**' to range over all individual constants of PL

DEFINITIONS:

- Expression of PL: A sequence of not necessarily distinct elements of the vocabulary of PL.
- Quantifier of PL: An expression of PL of the form (∀**x**) or (∃**x**). When a quantifier contains the variable ‘x’, it is known as an ‘x-quantifier’
- Atomic Formula of PL: An expression of PL that is an n-place predicate of PL followed by n individual terms of PL.

A FORMULA OF PL:

- A formula of PL is different from an expression of PL in the sense that a sentence of SL is different from a sequence of elements of the vocabulary of SL.
- So ‘(((((GHJH&v&’ is a sequence of elements of the vocabulary of SL, but is not a sentence of SL.
- Likewise, (((()(((a&bba) is an expression of PL, but not a formula of PL.

RECURSIVE DEFINITION OF ‘FORMULA OF PL’

1. Every atomic formula of PL is a formula of PL
2. If **P** is a formula of PL, so is ~**P**
3. If **P** and **Q** are formulae of PL, so are (**P** & **Q**), (**P** ∨ **Q**), (**P** ⊃ **Q**), and (**P** ≡ **Q**).
4. If **P** is a formula of PL that contains at least one occurrence of **x** and no **x**-quantifier, then (∀**x**)**P** and (∃**x**)**P** are both formulae of PL.
5. Nothing is a formula of PL unless it can be formed by repeated applications of clauses 1-4.

MORE DEFINITIONS:

- Logical Operator of PL: An expression of PL that is either a quantifier or a truth-functional connective.

SUBFORMULAE AND MAIN LOGICAL OPERATORS:

1. If **P** is an atomic formula of PL, then **P** contains no logical operator, and hence no main logical operator, and **P** is the only subformula of **P**.

2. If **P** is a formula of PL of the form ~**Q**, then the '~' that precedes **Q** is the main logical operator of **P**, and **Q** is the immediate subformula of **P**.

3. If **P** is a formula of PL of the form (**Q** & **R**), (**Q** ∨ **R**), (**Q** ⊃ **R**), or (**Q** ≡ **R**), then the binary connective between **Q** and **R** is the main logical operator of **P**, and **Q** and **R** are the immediate subformulae of **P**.

4. If **P** is a formula of PL of the form (∀**x**)**Q** or (∃**x**)**Q**, then the quantifier that occurs before **Q** is the main logical operator of **P**, while **Q** is the immediate subformula of **P**.

5. If **P** is a formula of PL, then every subformula of a subformula of **P** is a subformula of **P**, and **P** is a subformula of itself.

IS IT A FORMULA OF PL?

- (x)Px v Py

IS IT A FORMULA OF PL?

- (x)Px v Py

No.

While Px and Py are atomic formulae of PL, and so by clause 3 of our definition of ‘formula of PL’ can form Px v Py, nothing will let us form (x).

IS IT A FORMULA OF PL?

- $(\forall z)(\exists x)(Fzx \;\&\; Fxz)$

IS IT A FORMULA OF PL?

- (∀z)(∃x)(Fzx & Fxz)

  Yes

- ✓ Fzx and Fxz are both atomic formulae of PL, so (Fzx & Fxz) is a formula of PL (3).
- ✓ Since (Fzx & Fxz) is a formula of PL that contains x and contains no x-quantifier, (∃x)(Fzx & Fxz) is a formula of PL (4).
- ✓ Since (∃x)(Fzx & Fxz) is a formula of PL that contains z and no z-quantifier, (∀z)(∃x)(Fzx & Fxz) is a formula of PL (4).

IDENTIFY MAIN OPERATOR, BREAK INTO SUBFORMULAE, RINSE, REPEAT:

Formula	Subformulae	Main Operator	Type of Formula
(∃w)(Fw & ~Fw) ≡ (He & ~He)	Itself	≡	Truth-functional
	(∃w)(Fw & ~Fw)	(∃w)	Quantified
	(He & ~He)	&	Truth-functional
	(Fw & ~Fw)	&	Truth-functional
	~He	~	Truth-functional
	~Fw	~	Truth-functional
	Fw	None	Atomic
	He	None	Atomic