PHILOSOPHY 220

Symbolization in SL 2

Scope

- When we use connectives to join atomic sentences of *SL*, we must be concerned with the **scope** of the connectives we use. Parentheses () and Brackets [] help us to visually organize scope for molecular sentences in *SL*.
- Contrast
  - ~(A & B): It is not the case that both A and B
  - ~A & B: Both A is not the case and B is the case.
- The difference between the above is that the entire molecular sentence ‘A & B’ is in the scope of the negation, while in the sentence ‘~A & B’, only ‘A’ is in the scope of the negation.

Logic is not math!!!

- While '~' certainly looks like '-', and while 'negation' and 'negative' sound like they ought to have a great deal to do with one another, resist the temptation to treat the logical negation symbol like the mathematical negative symbol.
- Example:
  - Does -(3 + 5) = -3 + -5?
  - Is ~(A & B) truth-functionally equivalent to ~A & ~B?

- Let's Check:

Equivalence on a Truth Table

Ref.		First Sent.			Second Sent.		
A	B	~	A & B		~A	&	~B
T	T						
T	F						
F	T						
F	F						

Equivalence on a Truth Table

Ref.		First Sent.			Second Sent.		
A	B	~	A & B		~A	&	~B
T	T		T				
T	F		F				
F	T		F				
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F	T	T	F		T		
F	F	T	F		T		

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T	F	T	F		F		T
F	T	T	F		T		F
F	F	T	F		T		T

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A	B	~	A & B		~A	&	~B
T	T	F	T		F	F	F
T	F	T	F		F	F	T
F	T	T	F		T	F	F
F	F	T	F		T	T	T

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Ref.		First Sent.			Second Sent.		
A	B	~	A & B		~A	&	~B
T	T	**F**	T		F	**F**	F
T	F	**T**	F		F	**F**	T
F	T	**T**	F		T	**F**	F
F	F	**T**	F		T	**T**	T

‘~ (A & B)’ is not logically equivalent to ‘~A & ~B’ because they do not have the same truth values in the same circumstances.

Truth Functionality Illustrated:

- Consider the molecular sentence :
- (A & B) v [(~B v A) & (~A v B)]
- Now assume A is true and B is false. What is the truth value of the whole sentence?

(A & B) v [(~B v A) & (~A v B)]

(T & F) v [(~F v T) & (~T v F)]

F v [(~F v T) & (~T v F)]

F v [(T v T) & (F v F)]

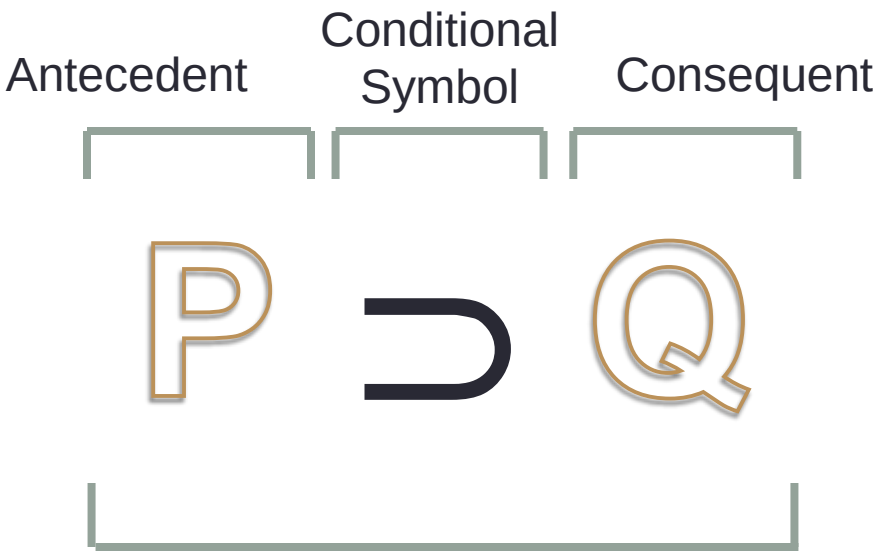
F v [T & (F v F)]

F v [T & F]

F v F

F

The Material Conditional



Material Conditional Definition

P	Q	$P \supset Q$
T	T	
T	F	
F	T	
F	F	

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P	Q	$P \supset Q$
T	T	T
T	F	
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Very Straightforward. “If you clean the barn then I pay you five bucks.” is true when it is true that you clean the barn and when it is true that I pay you five bucks.

Material Conditional Definition

P	Q	$P \supset Q$
T	T	T
T	F	F
F	T	
F	F	

Also Straightforward. “If you clean the barn then I pay you five bucks.” is false when it is true that you clean the barn and false that I pay you five bucks.

Material Conditional Definition

P	Q	$P \supset Q$
T	T	T
T	F	F
F	T	T
F	F	T

A bit counterintuitive: “If you clean the barn then I pay you five bucks.” is true whenever it is not false. If the antecedent is false (you do not clean out the barn) then the conditional will not be falsified, and will be counted as true. Whether I give you five bucks or not, I still haven’t lied to you.

Material Conditional Equivalence

- Consider whether the following are logically equivalent:
  - “If you clean the barn I’ll pay you $5.”
  - “Either you don’t clean the barn, or I’ll pay you $5”
- The preceding are symbolized:
  - $C \supset P$
  - $\sim C \vee P$

Material Conditional Equivalence

P	Q	~P	v	Q
T	T	F	**T**	T
T	F	F	**F**	F
F	T	T	**T**	T
F	F	T	**T**	F

Material Conditional Equivalence

P	Q	~P	v	Q	P ⊃ Q
T	T	F	**T**	T	**T**
T	F	F	**F**	F	**F**
F	T	T	**T**	T	**T**
F	F	T	**T**	F	**T**

Material Conditional Equivalence

- Many students want to make a conditional false when the antecedent is false. That would make the symbol '⊃' mean the same thing as the '&'.
- Does 'If P then Q' mean the same thing as 'P and Q'?
- Clearly not. The person who utters the latter is asserting the truth of both P and Q while the person who utters the former is asserting neither the truth nor falsity of either P or Q.
- The material conditional asserts a relationship between P and Q that is false when the antecedent (P) is true while the consequent (Q) is false, and true otherwise.

Material Conditionals in Arguments

- Further, if the material conditional is not defined as it is, then some obviously valid argument forms come out funny (specifically, modus ponens looks like it has a superfluous premise and modus tollens is only trivially valid).
- See me later for a fuller explanation of that point.

Material Conditional and the English 'If…Then…'

- Many uses of "If…Then…" in English are not instances of the material conditional.
- Consider the truth value of: "If there is an Elephant in the room, then it is raining."

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- Consider the truth value of: "If there is an Elephant in the room, then it is raining."
  - The above is true (barring an elephant being in the room and clear weather when I present these notes)
  - If you think it must be false, you are reading it as a causal conditional, which is a material conditional with extra baggage. In a causal conditional "If P then Q" means "P causes Q"

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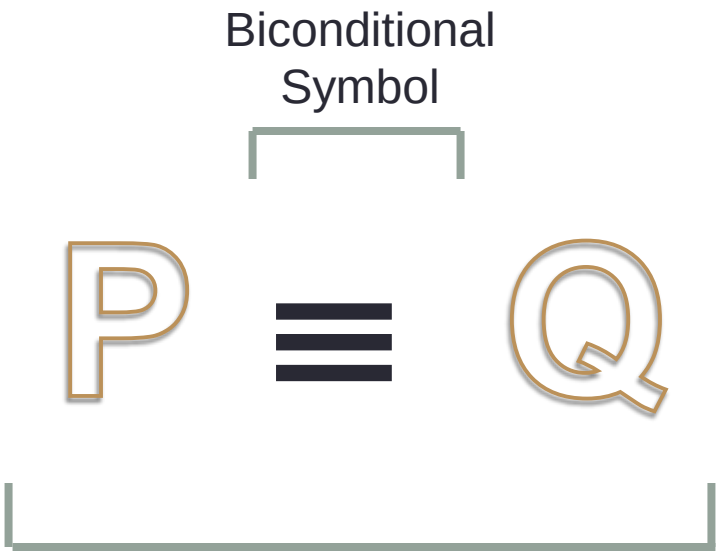
- Many uses of "If…Then…" in English are not instances of the material conditional.
- Consider symbolizing: "If the Germans had won the second world war, then everyone would speak German"
  - Notice that there are not *two* propositions expressed because 'the Germans *had* won…' does not express a proposition by itself, nor does 'everyone *would* speak German'.
  - This is a counterfactual, or subjunctive conditional. It is best symbolized 'P'.

The Material Biconditional



Material Biconditional Definition

P	Q	P ≡ Q
T	T	T
T	F	F
F	T	F
F	F	T

Material Biconditional and ‘=‘

- The biconditional is a sign of logical equivalence and not general equivalence or identity.
- The sentence ‘P ⊃ Q’ is logically equivalent to the sentence ‘~P v Q’ but is not *the same sentence*.
- So ‘(P ⊃ Q) ≡ (~P v Q)’ is a tautology while

  ‘(P ⊃ Q) = (~P v Q)’ is false