PHILOSOPHY 220

Symbolization in SL 2

Scope

- When we use connectives to join atomic sentences of *SL*, we must be concerned with the **scope** of the connectives we use. Parentheses () and Brackets [] help us to visually organize scope for molecular sentences in *SL*.
- Contrast
  - ~(A & B): It is not the case that both A and B
  - ~A & B: Both A is not the case and B is the case.
- The difference between the above is that the entire molecular sentence ‘A & B’ is in the scope of the negation, while in the sentence ‘~A & B’, only ‘A’ is in the scope of the negation.

Logic is not math!!!

- While ‘~’ certainly looks like ‘-’, and while ‘negation’ and ‘negative’ sound like they ought to have a great deal to do with one another, resist the temptation to treat the logical negation symbol like the mathematical negative symbol.
- Example:
  - Does -(3 + 5) = -3 + -5?
  - Is ~(A & B) truth-functionally equivalent to ~A & ~B?
- Let’s Check:

Equivalence on a Truth Table

Ref.		First Sent.			Second Sent.		
A	B	~	A & B		~A	&	~B
T	T						
T	F						
F	T						
F	F						

Equivalence on a Truth Table

Ref.		First Sent.			Second Sent.		
A	B	~	A & B		~A	&	~B
T	T		T				
T	F		F				
F	T		F				
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T	F	T	F		F		
F	T	T	F		T		
F	F	T	F		T		

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A	B	~	A & B		~A	&	~B
T	T	F	T		F		F
T	F	T	F		F		T
F	T	T	F		T		F
F	F	T	F		T		T

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A	B	~	A & B		~A	&	~B
T	T	F	T		F	F	F
T	F	T	F		F	F	T
F	T	T	F		T	F	F
F	F	T	F		T	T	T

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Ref.		First Sent.			Second Sent.		
A	B	~	A & B		~A	&	~B
T	T	**F**	T		F	**F**	F
T	F	**T**	F		F	**F**	T
F	T	**T**	F		T	**F**	F
F	F	**T**	F		T	**T**	T

'~ (A & B)' is not logically equivalent to '~A & ~B' because they do not have the same truth values in the same circumstances.

Truth Functionality Illustrated:

- Consider the molecular sentence :
- (A & B) v [(~B v A) & (~A v B)]
- Now assume A is true and B is false. What is the truth value of the whole sentence?

(A & B) v [(~B v A) & (~A v B)]

(T & F) v [(~F v T) & (~T v F)]

F v [(~F v T) & (~T v F)]

F v [(T v T) & (F v F)]

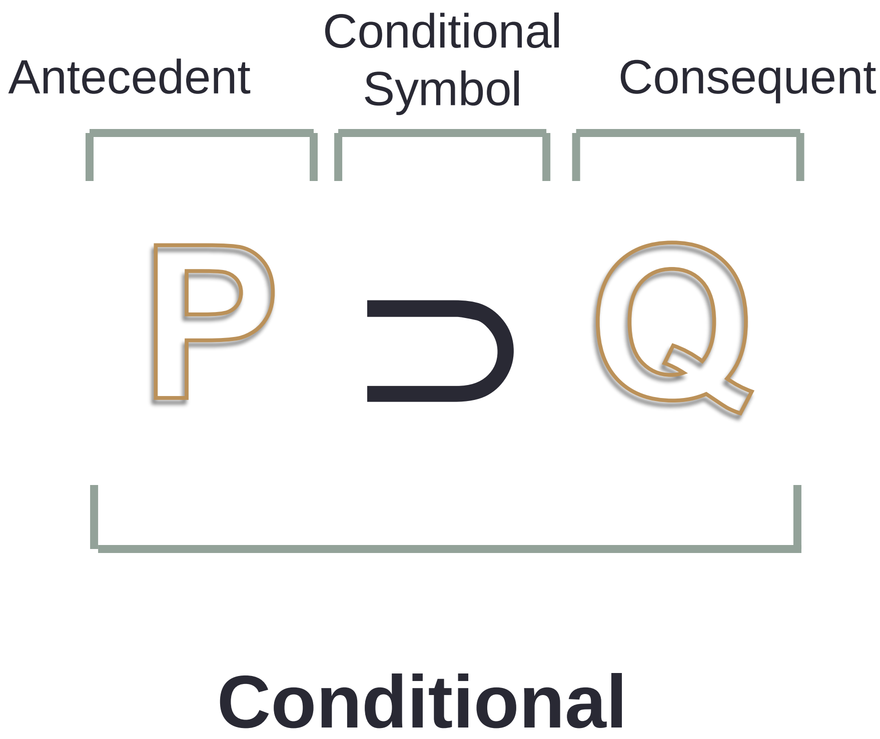
F v [T & (F v F)]

F v [T & F]

F v F

F

The Material Conditional



Material Conditional Definition

P	Q	P ⊃ Q
T	T	
T	F	
F	T	
F	F	

Material Conditional Definition

P	Q	P ⊃ Q
T	T	T
T	F	
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Very Straightforward. “If the pitcher throws a fastball, then the batter hits a home run.” is true when it is true that the pitcher throws a fastball and true that the batter hits a home run.

Material Conditional Definition

P	Q	P ⊃ Q
T	T	T
T	F	F
F	T	
F	F	

Also Straightforward. “If the pitcher throws a fastball, then the batter hits a home run.” is false when it is true that the pitcher throws a fastball and false that the batter hits a home run.

Material Conditional Definition

P	Q	P ⊃ Q
T	T	T
T	F	F
F	T	T
F	F	T

A bit counterintuitive: "If the pitcher throws a fastball, then the batter hits a home run." is true whenever it is not false. If the antecedent is false (if the pitcher does not throw a fastball) then the conditional will not be falsified, and will be counted as true.

Material Conditional Equivalence

- Consider whether the following are logically equivalent:
  - “If you clean the barn I’ll pay you $5.”
  - “Either you don’t clean the barn, or I’ll pay you $5”
- The preceding are symbolized:
  - $C \supset P$
  - $\sim C \vee P$

Material Conditional Equivalence

P	Q	~P	v	Q
T	T	F	**T**	T
T	F	F	**F**	F
F	T	T	**T**	T
F	F	T	**T**	F

Material Conditional Equivalence

P	Q	~P	v	Q	P ⊃ Q
T	T	F	**T**	T	**T**
T	F	F	**F**	F	**F**
F	T	T	**T**	T	**T**
F	F	T	**T**	F	**T**

Material Conditional Equivalence

- Many students want to make a conditional false when the antecedent is false. That would make the symbol ‘⊃’ mean the same thing as the ‘&’.
- Does ‘If P then Q’ mean the same thing as ‘P and Q’?
- Clearly not. The person who utters the latter is asserting the truth of both P and Q while the person who utters the former is asserting neither the truth nor falsity of either P or Q.
- The material conditional asserts a relationship between P and Q that is false when the antecedent (P) is true while the consequent (Q) is false, and true otherwise.

Material Conditionals in Arguments

- Further, if the material conditional is not defined as it is, then some obviously valid argument forms come out funny (specifically, modus ponens looks like it has a superfluous premise and modus tollens is invalid).
- See me later for a fuller explanation of that point.

Material Conditional and the English ‘If…Then…’

- Many uses of “If…Then…” in English are not instances of the material conditional.
- Consider the truth value of: “If there is an Elephant in the room, then it is raining.”

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- Consider the truth value of: “If there is an Elephant in the room, then it is raining.”
  - The above is true (barring an elephant being in the room and clear weather when I present these notes)
  - If you think it must be false, you are reading it as a causal conditional, which is a material conditional with extra baggage. In a causal conditional “If P then Q” means “P causes Q”

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Material Conditional and the English ‘If…Then…’

- Many uses of “If…Then…” in English are not instances of the material conditional.
- Consider symbolizing: “If the Germans had won the second world war, then everyone would speak German”
  - Notice that there are not *two* propositions expressed because ‘the Germans *had* won…’ does not express a proposition by itself, nor does ‘everyone *would* speak German’.
  - This is a counterfactual, or subjunctive conditional. It is best symbolized ‘P’.

The Material Biconditional

Biconditional
Symbol

$$P \equiv Q$$

Biconditional

Material Biconditional Definition

P	Q	P ≡ Q
T	T	T
T	F	F
F	T	F
F	F	T

Material Biconditional and ‘=‘

- The biconditional is a sign of logical equivalence and not general equivalence or identity.
- The sentence ‘P ⊃ Q’ is logically equivalent to the sentence ‘~P v Q’ but is not *the same sentence*.
- So ‘(P ⊃ Q) ≡ (~P v Q)’ is logically true while

  ‘(P ⊃ Q) = (~P v Q)’ is false