Philosophy 220

Truth Functional Properties Expressed in terms of Consistency

The semantic concepts of truth-functional logic:

* Tautology
* Contradiction
* Contingency
* Entailment
* Validity
* Equivalence
* Consistency

The concepts of truth-functional logic:

* The section of the text pp. 110-113 aims to demonstrate that all of the semantic concepts of truth-functional logic can be explained in terms of consistency.
* As it happens, all of the semantic concepts of truth-functional logic can be explained in terms of any of the other semantic concepts of truth-functional logic listed previously.

Why Consistency?

* If all of the other semantic concepts of truth-functional logic can be explained via consistency, then a system that tests for consistency can test for all of the other concepts as well.
* We will be replacing truth-tables with a system based on testing for consistency (but that is much easier to learn if you already are very familiar with truth-tables).
* This new system, called the 'semantic tree system' will be our primary system for determining validity, entailment, equivalency, etc. for the remainder of the course.

Consistency (Review)

* A set of sentences of SL is consistent if and only if there is at least one truth value assignment [of the constituents of the set of sentences] on which all the members of the set are true.

Contradiction

Definition

* A sentence of SL is a contradiction if and only if it is false on every possible truth-value assignment of its constituents.

Explained via consistency

* A sentence **P** is truth-functionally false if and only if {**P**} is truth-functionally inconsistent.
* Since inconsistent sets are sets that can never all be true at the same time, and since the unit set of **P** has only one member, it must always be false to be inconsistent.

Tautology

Definition

* A sentence of SL is a tautology if and only if it is true on every possible truth-value assignment of its constituents.

Explained via Consistency

* A sentence **P** is a tautology if and only if {~**P**} is truth-functionally inconsistent.
* The only member of any inconsistent set is a contradiction, and the negation of a contradiction is a tautology, so if ~**P** is a contradiction, then **P** is a tautology.

Contingency

Definition

* A sentence of SL is contingent if and only if it is neither a tautology nor a contradiction.

Explained via consistency

* A sentence **P** is truth-functionally indeterminate if and only if both {~**P**} and {**P**} are truth-functionally consistent.
* If the above are consistent, then **P** is neither a tautology nor a contradiction.

Equivalence

Definition

* Sentences P and Q of SL are equivalent if and only if there is no truth value assignment [for the components of P and Q] on which P and Q have different truth-values.

Explained via consistency

* Sentences P and Q of SL are equivalent if and only if {~(P ≡ Q)} is inconsistent
* If P and Q have the same truth values, P ≡ Q is a tautology. That would mean that ~(P ≡ Q) would be a contradiction, and so would make for an inconsistent set.

A new symbol:

* To define validity and entailment by means of consistency, it is useful to introduce a new symbol:
* '$\cup$' is the union symbol.
* The union symbol is used to express the combination of two sets together.
* Example: $\{A, B, C\} \cup \{D\}$ is $\{A, B, C, D\}$

Entailment

Definition

* A set Γ of sentences of SL entails a sentence **P** if and only if there is no truth-value assignment on which every member of Γ is true and **P** is false.

Explained via Consistency

* $\Gamma \models$ **P** if and only if $\Gamma \cup \{\sim$**P**$\}$ is truth-functionally inconsistent.
* Next slide contains a more detailed rationale…

$\Gamma \models$ **P** if and only if $\Gamma \cup \{\sim$**P**$\}$ is inconsistent.

* If the set Γ entails **P**, then there is no truth-value assignment that makes the members of Γ true while **P** is false. That means that whenever the members of Γ are all true, **P** is true also, so $\Gamma \cup \{\sim$**P**$\}$ would be inconsistent.
* Side note: If Γ is inconsistent to begin with, then $\Gamma \cup \{\sim$**P**$\}$ is still inconsistent, and Γ still entails **P**, because inconsistent sets entail anything.

Validity

* Since validity is simply a special case of entailment, the same procedure can demonstrate that validity, like entailment, can be described in terms of consistency.
* If an argument is valid, then the union of the set of its premises and the negation of its conclusion will form an inconsistent set.