

Proof of DeMorgan's Law 1
(Proof that $\sim(P \& Q) \equiv (\sim P \vee \sim Q)$ is a theorem)

Assumptions in force:	Line Number:	Sentence of SL:	Justification:	Notes:
1a	1	$\sim(P \& Q)$	A1a	for \equiv l, get $\sim P \vee \sim Q$
2 1a	2	$\sim(\sim P \vee \sim Q)$	A2	for \sim l
3 2 1a	3	P	A3	for \sim l
4 3 2 1a	4	Q	A4	for \sim l
4 3 2 1a	5	P & Q	3,4 &l	
4 3 2 1a	6	(P & Q) & $\sim(P \& Q)$	1,,5 &l	contradiction
! 1a	7	$\sim Q$	4-6 \sim l	
3 2 1a	8	$\sim P \vee \sim Q$	7 vI	
3 2 1a	9	$(\sim P \vee \sim Q) \& \sim(\sim P \vee \sim Q)$	2,8 &l	contradiction
! 2 1a	10	$\sim P$	3-9 \sim l	
2 1a	11	$\sim P \vee \sim Q$	10 vI	
2 1a	12	$(\sim P \vee \sim Q) \& \sim(\sim P \vee \sim Q)$	2,11 &l	contradiction
! 1a	13	$\sim\sim(\sim P \vee \sim Q)$	1-12 \sim l	
1a	14	$\sim P \vee \sim Q$	13 \sim E	
1b	15	$\sim P \vee \sim Q$	A1b	second half of \equiv l, get $\sim(P \& Q)$
5a 1b	16	$\sim P$	A5a	for vE, to get $\sim(P \& Q)$
6 5a 1b	17	P & Q	A6	for \sim l
6 5a 1b	18	P	17 &E	
6 5a 1b	19	P & $\sim P$	16,18 &l	contradiction
! 5a 1b	20	$\sim(P \& Q)$	17-19 \sim l	
5b 1b	21	$\sim Q$	A5b	second half of vE, to get $\sim(P \& Q)$
7 5b 1b	22	P & Q	A7	for \sim l
7 5b 1b	23	Q	22 &E	
7 5b 1b	24	Q & $\sim Q$	21,24 &l	contradiction
! 5b 1b	25	$\sim(P \& Q)$	22-24 \sim l	
! 1b	26	$\sim(P \& Q)$	15,16-20,21-25 vE	
!	27	$\sim(P \& Q) \equiv (\sim P \vee \sim Q)$	1-14,15-26 \equiv I	QED

Proof of DeMorgan's Law 2
(Proof that ' $\sim(P \vee Q) \equiv (\sim P \ \& \ \sim Q)$ ' is a theorem)

Assumptions in force:	Line Number:	Sentence of SL:	Justification:	Notes:
1a	1	$\sim(P \vee Q)$	A1a	for \equiv I, get $(\sim P \ \& \ \sim Q)$
2 1a	2	P	A2	for \sim I
2 1a	3	$P \vee Q$	2, vI	
2 1a	4	$(P \vee Q) \ \& \ \sim(P \vee Q)$	1,4 &I	contradiction
! 1a	5	$\sim P$	2-4 \sim I	
3 1a	6	Q	A3	for \sim I
! 1a	7	$P \vee Q$	6 vI	
3 1a	8	$(P \vee Q) \ \& \ \sim(P \vee Q)$	1,7 &I	contradiction
! 1a	9	$\sim Q$	6-8 \sim I	
1a	10	$\sim P \ \& \ \sim Q$	5,9 &I	
1b	11	$\sim P \ \& \ \sim Q$	A1b	Second half of \equiv I, get $\sim(P \vee Q)$
4 1b	12	$P \vee Q$	A4	for \sim I
5a 4 1b	13	P	A5a	for vE, get R & $\sim R$
6 5a 4 1b	14	$\sim(R \ \& \ \sim R)$	A6	for \sim I
6 5a 4 1b	15	$\sim P \ \& \ \sim Q$	11 R	
6 5a 4 1b	16	$\sim P$	15 &E	
6 5a 4 1b	17	$P \ \& \ \sim P$	13,16 &I	contradiction
! 5a 4 1b	18	$\sim\sim(R \ \& \ \sim R)$	14-17 \sim I	
5a 4 1b	19	$(R \ \& \ \sim R)$	18 \sim E	
5b 4 1b	20	Q	A5b	second half of vE, get R & $\sim R$
7 5b 4 1b	21	$\sim(R \ \& \ \sim R)$	A7	for \sim I
7 5b 4 1b	22	$\sim P \ \& \ \sim Q$	11 R	
7 5b 4 1b	23	$\sim Q$	22 &E	
7 5b 4 1b	24	$Q \ \& \ \sim Q$	20,23 &I	contradiction
! 5b 4 1b	25	$\sim\sim(R \ \& \ \sim R)$	21-24 \sim I	
5b 4 1b	26	$R \ \& \ \sim R$	25 \sim E	
! 4 1b	27	$R \ \& \ \sim R$	12,13-19,20-26 vE	contradiction
! 1b	28	$\sim(P \vee Q)$	12-27 \sim I	
!	29	$\sim(P \vee Q) \equiv (\sim P \ \& \ \sim Q)$	1-10,11-28 \equiv I	QED

Proof of Contraposition (transposition)

Assumptions in force:	Line Number:	Sentence of SL:	Justification:	Notes:
	1	$P \supset Q$	Premise	Derive: $\sim Q \supset \sim P$
1	2	$\sim Q$	A1	for \supset I
2 1	3	P	A2	for \sim I
2 1	4	Q	1,3 \supset E	
2 1	5	$Q \ \& \ \sim Q$	2,4 $\&$ I	contradiction
! 1	6	$\sim P$	3-5 \sim I	
!	7	$\sim Q \supset \sim P$	2-6 \supset I	QED

Proof of Distribution 1

Assumptions in force:	Line Number:	Sentence of SL:	Justification:	Notes:
	1	$P \& (Q \vee R)$	Premise	Derive: $(P \& Q) \vee (P \& R)$
1	2	$\sim[(P \& Q) \vee (P \& R)]$	A1	for $\sim I$
2 1	3	$P \& Q$	A2	for $\sim I$
2 1	4	$(P \& Q) \vee (P \& R)$	3 $\vee I$	
2 1	5	$\sim[(P \& Q) \vee (P \& R)] \& [(P \& Q) \vee (P \& R)]$	2,4 $\& I$	contradiction
! 1	6	$\sim(P \& Q)$	3-5 $\sim I$	
! 1a	7	$P \& R$	A3	for $\sim I$
3 1	8	$(P \& Q) \vee (P \& R)$	7 $\vee I$	
3 1	9	$\sim[(P \& Q) \vee (P \& R)] \& [(P \& Q) \vee (P \& R)]$	2,8 $\& I$	contradiction
! 1	10	$\sim(P \& R)$	7-9 $\sim I$	
4 1	11	Q	A4	for $\sim I$
4 1	12	P	1 $\& E$	
4 1	13	$P \& Q$	11,12 $\& I$	
4 1	14	$(P \& Q) \& \sim(P \& Q)$	6,13 $\& I$	contradiction
! 1	15	$\sim Q$	11-14 $\sim I$	
5 1	16	R	A5	for $\sim I$
5 1	17	P	1 $\& E$	
5 1	18	$P \& R$	16,17 $\& I$	
5 1	19	$(P \& R) \& \sim(P \& R)$	10,19 $\& I$	contradiction
! 1	20	$\sim R$	16-19 $\sim I$	
1	21	$\sim Q \& \sim R$	15,20 $\& I$	
6 1	22	$Q \vee R$	A6	for $\sim I$
7a 6 1	23	Q	A7a	for $\vee E$ (get $S \& \sim S$)
8 7a 6 1	24	$\sim(S \& \sim S)$	A8	for $\sim I$
8 7a 6 1	25	$\sim Q$	21 $\& E$	
8 7a 6 1	26	$Q \& \sim Q$	23,25 $\& I$	contradiction
! 7a 6 1	27	$\sim\sim(S \& \sim S)$	24-26 $\sim I$	
7a 6 1	28	$S \& \sim S$	27 $\sim E$	
7b 6 1	29	R	A7b	second half of $\vee E$ (get $S \& \sim S$)
9 7b 6 1	30	$\sim(S \& \sim S)$	A9	for $\sim I$
9 7b 6 1	31	$\sim R$	21 $\& E$	
9 7b 6 1	32	$R \& \sim R$	29,31 $\& I$	contradiction
! 7b 6 1	33	$\sim\sim(S \& \sim S)$	30-32 $\sim I$	
7b 6 1	34	$S \& \sim S$	33 $\sim E$	
! 6 1	35	$S \& \sim S$	22,23-28,29-34 $\vee E$	contradiction
! 1	36	$\sim(Q \vee R)$	22-35 $\sim I$	
1	37	$Q \vee R$	1, $\& E$	
1	38	$\sim(Q \vee R) \& (Q \vee R)$	36,37 $\& I$	contradiction
!	39	$\sim\sim[(P \& Q) \vee (P \& R)]$	2-38 $\sim I$	
	40	$(P \& Q) \vee (P \& R)$	39 $\sim E$	QED

Proof of Distribution 2

Assumptions in force:	Line Number:	Sentence of SL:	Justification:	Notes:
	1	$(P \ \& \ Q) \vee (P \ \& \ R)$	Premise	Derive: $P \ \& \ (Q \vee R)$
1a	2	$P \ \& \ Q$	A1a	for $\vee E$ (get $[P \ \& \ (Q \vee R)]$)
1a	3	Q	2 &E	
1a	4	$Q \vee R$	3 $\vee I$	
1a	5	P	2 &E	
1a	6	$P \ \& \ (Q \vee R)$	4,5 &I	
! 1a	7	$P \ \& \ R$	A1b	second half of $\vee E$ (get $[P \ \& \ (Q \vee R)]$)
1b	8	R	7 &E	
1b	9	$Q \vee R$	8 $\vee I$	
1b	10	P	7 &E	
1b	11	$P \ \& \ (Q \vee R)$	9,10 &I	
!	12	$P \ \& \ (Q \vee R)$	1,2-6,7-11 $\vee E$	QED