Philosophy 220

Quantifiers

Awkwardness of quantity terms

- You may have noticed that sentences involving quantity terms like ‘some’ involved strings of disjunctions and that sentences involving quantity terms like ‘every’ involve strings of conjunctions.
- To abbreviate this awkwardness, we’ll introduce two new symbols to PL that will allow us to use universal quantity terms and particular quantity terms more economically.

The Universal Quantifier, $(\forall x)$

- The above symbol is used whenever one wishes to quantify over a whole class of things (when words like 'all', 'every', 'only' etc. are used).
- The symbol is most straightforwardly read in English as "For all of x…" so when P stands for the predicate "likes pie", the PL sentence '$(\forall x)Px$' should be read into English as "For all of x, x likes pie"
- The above means something like "Everyone likes pie" for everyone that we are talking about (the UD).

The Existential Quantifier, (∃x)

- The above symbol is used whenever one wishes to quantify over particular things (when words like 'some', 'someone', 'at least one' etc. are used).
- The symbol is most straightforwardly read in English as "There is an x such that…" so when P' stands for the predicate "likes pie", the PL sentence '(∃x)Px' should be read into English as "There is an x such that x likes pie"
- The above means something like "Someone likes pie" out of everyone that we are talking about (the UD).

Quantifiers and negations

- This will serve to confuse you if you aren't very careful.
- Consider the predicate L" to be 'loves' and the constant 'a' to stand for Alan:
- $(\forall x)$ ~Lxa (Nobody loves Alan (For all x, it is not the case that x loves Alan))
- $(\forall x)$ ~Lax (Alan loves nobody (For all x, it is not the case that Alan loves x))
- ~$(\forall x)$Lxa (Not everyone loves Alan (It is not the case that for all x, x loves Alan))
- ~$(\forall x)$Lax (Alan doesn't love everyone (It is not the case that for all x, Alan loves x))

Quantifiers and negations (cont.)

- ~(∃x)Lax (Alan loves nobody (It is not the case that there is an x that Alan loves))
- ~(∃x)Lxa (Nobody loves Alan (It is not the case that there is an x such that x loves Alan))
- (∃x) ~Lax (There is someone that Alan doesn't love (There is an x such that it is not the case that Alan loves x))
- (∃x) ~Lxa (Someone doesn't love Alan (There is an x such that it is not the case that x loves Alan))

Equivalence for quantifiers:

~(∃x) ~Lax: "It is not the case that there is an x such that it is not the case that Alan loves x"

- In other words, "There is nobody that Alan does not love", or "Alan loves everyone".
- So '~(∃x)~Px' is equivalent to '(∀x)Px'.
- Exercise for the alert student:
- Is '~(∀x)~Px' is equivalent to '(∃x)Px'?
- '~(∃x)Px' is equivalent to '(∀x) ~Px'?