| | | | | |
|---|---|---|---|---|
| | 1 | ~(F ⊃ G) | Given | |
| | 2 | ~(G ⊃ H) | Given | Derive: I |
| 1 | 3 | ~I | A1 | ~I (contradiction) |
| 2 1 | 4 | G | A2 | ~I (contradiction) |
| 3 2 1 | 5 | F | A3 | ⊃I (G) |
| 3 2 1 | 6 | G | 4 R | |
| ! 2 1 | 7 | F ⊃ G | 5-6 ⊃I | |
| 2 1 | 8 | (F ⊃ G) & ~(F ⊃ G) | 1,7 &I | contradiction |
| ! 1 | 9 | ~G | 4-8 ~I | Note: ~H ⊃~G is equivalent to G ⊃ H |
| 4 1 | 10 | ~H | A4 | ⊃I (~G) |
| 4 1 | 11 | ~G | 9 R | |
| ! 1 | 12 | ~H ⊃~G | 10-11 ⊃I | See note on 9 |
| 5 1 | 13 | G | A5 | ⊃I (H) |
| 6 5 1 | 14 | ~H | A6 | ~I (contradiction) |
| 6 5 1 | 15 | ~G | 12,14 ⊃E | |
| 6 5 1 | 16 | G & ~G | 13,15 &I | contradiction |
| ! 5 1 | 17 | ~~H | 14-16 ~I | |
| 5 1 | 18 | H | 17 ~E | |
| ! 1 | 19 | G ⊃ H | 13-18 ⊃I | |
| 1 | 20 | (G ⊃ H) & ~(G ⊃ H) | 2,19 &I | Contradiction |
| ! | 21 | ~~I | 3-20 ~I | |
| | 22 | I | 21 ~E | QED |

| | | | |
|---|---|---|---|
| i | M. | **P** | Ai |
| i | N. | **Q** & ~**Q** | - |
| ! | N+1 | ~**P** | M-N ~I |

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| i | M. | **P** | Ai |
| i | N. | **Q** & ~**Q** | - |
| ! | N+1 | ~**P** | M-N ~I |

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| i | M. | **P** | Ai |
| i | N. | **Q** | - |
| ! | N+1 | **P** ⊃ **Q** | M-N ⊃I |

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| i | M. | **P** | Ai |
| i | N. | **Q** | - |
| ! | N+1 | **P** ⊃ **Q** | M-N ⊃I |

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| i | M. | **P** | Ai |
| i | N. | **Q** | - |
| ! | N+1 | **P** ⊃ **Q** | M-N ⊃I |

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| i | M. | **P** | Ai |
| i | N. | **Q** & ~**Q** | - |
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| 6 5 1 | 16 | G & ~G | 13,15 &I | contradiction |
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| 5 1 | 18 | H | 17 ~E | |
| ! 1 | 19 | G ⊃ H | 13-18 ⊃I | |
| 1 | 20 | (G ⊃ H) & ~(G ⊃ H) | 2,19 &I | Contradiction |
| ! | 21 | ~~I | 3-20 ~I | |
| | 22 | I | 21 ~E | QED |

| | | | |
|---|---|---|---|
| i…k | M. | **P** | |
| i…k, l…n | N. | **Q** | |
| i…k, l…n | N+1. | **P** | M, R |

| | | |
|---|---|---|
| L. | **P** | |
| M. | **Q** | |
| N. | **P** & **Q** (or **Q** & **P**) | L,M &I |

| | | |
|---|---|---|
| L. | **P** ⊃ **Q** | |
| M. | **P** | |
| N. | **Q** | L,M ⊃E |

| | | |
|---|---|---|
| L. | **P** | |
| M. | **Q** | |
| N. | **P** & **Q** (or **Q** & **P**) | L,M &I |

M. ~~P

N. P M ~E