Philosophy 220

Identity, Definite Descriptions, Properties of Relations, and Functions

Identity:

- Sometimes we wish to express when two things are identical or non-identical.
- We could do this in PL by specifying a two-place predicate '_____ is identical with _____'.
- We could also introduce a familiar symbol to express identity (=) and incorporate it into PL as a permanent two-place predicate that expresses the identity relation.
  - In PL (now PLE) then, 'x=y' is a two place predicate that establishes that x is identical with y.
  - '~x=y' indicates that it is not the case that x is identical with y.

Sentences with an identity predicate:

- Everyone has seen Star Wars except Erica.
  - UD: People
  - Sx: x has seen Star Wars
  - e: Erica
    - $\sim Se \ \& \ (\forall x)(\sim e=x \supset Sx)$
- There are at most two pears in the basket:
  - Px: x is a pear
  - Nxy: x is in y
  - b: basket
    - $(\forall x)(\forall y)(\forall z)[([(Px \ \& Py) \ \& \ Pz] \ \& \ [(Nxb \ \& \ Nyb) \ \& \ Nzb]) \supset ((z=x \ v \ z=y)]$

Definite Descriptions

- Remember that there are two kinds of singular terms that we symbolize as lower-case letters in PL:
  - Proper Names
  - Definite Descriptions
- So far we have only used proper names as lower-case letters.
- We may now get at the structure of definite descriptions now that we have the identity predicate.
- (Historical Note: This treatment of definite descriptions is due to Bertrand Russell)

The structure of Definite Descriptions:

- Since definite descriptions are specifically designed to pick only one thing out of the world, it seems intuitive to think that definite descriptions would make use of the identity predicate.
- Consider: "Fred Durst was a member of the worst band in the world"
  - UD: People and Bands
  - Mxy: x was a member of y
  - Wx: x is the worst band in the world
  - d: Fred Durst
  - $(\exists y)((Mdy) \& (\forall x)(Wx \supset x=y))$

Another Definite Description:

- UD: Everyone
- Bx: x is bald
- Fx: x is the present King of France
- "The present King of France is bald"
  - $(\exists x)[(Fx \ \& \ (\forall y)(Fy \supset x=y)) \ \& \ Bx]$
  - The above is false because no entity satisfies the definite description that is the existentially quantified first conjunct.

Properties of Relations

- Every multi-place predicate expresses a relation between *n* things, where *n* is the number of places in the predicate.
- There are different properties that some relations have and that others do not:
  - Transitivity: Where A is any transitive relation, the following sentence of PL should be true:
    - $(\forall x)(\forall y)(\forall z)[(Axy \ \& \ Ayz) \supset Axz)]$
  - Symmetry: Where A is any symmetric relation, the following sentence of PL should be true:
    - $(\forall x)(\forall y)(Axy \equiv Ayx)$
  - Reflexivity: Where A is any reflexive relation, the following sentence of PL should be true:
    - $(\forall x)(Axx)$

Relations that are:

- Transitive, Symmetric, and Reflexive:
  - Identity, being in the same place as, being the same age as…
- Symmetric and Reflexive:
  - Has met…
- Transitive and Reflexive:
  - Contains, is less than or equal to…
- Transitive and Symmetric:
  - Is a sibling of…
- Transitive:
  - Is heavier than, is taller than…
- Symmetric:
  - Is not equal to…
- Reflexive:
  - x knows what y is thinking
- None of these:
  - Loves

Functions:

- A function is an operation that takes one or more elements of a set as inputs and returns a single value.
- Each of the truth-functional connectives are functions.
  - ~ returns a value of F when T is input, and returns a value of T when F is input.
  - & returns a value of T when TT is input, and a value of F when TF, FT, or FF is input.
  - v returns a value of F when FF is input, and a value of T when TT, TF, or FT is input.
  - ⊃ returns a value of F when TF is input, and a value of T when TT, FT, or FF is input.
  - ≡ returns a value of T when TT or FF is input, and a value of F when TF or FT is input.

FUNCTORS:

- In our notation, we will use an italicized lowercase letter *a-z* with one or more primes (') after it as a functor.
  - So *a'* is a functor with a singular input (the text uses the term 'argument' instead of 'input').
  - *b"* is a functor with two inputs (and so on, there is no formal limit to the number of inputs a function has).
- Functors have the following rules:
  1. An **n**-place function must yield one and only one value for each **n**-tuple of inputs, where an **n**-tuple is an ordered set containing **n** members.
  2. The value of a function for an **n**-tuple of members of a UD must be a member of that UD.

Complex Terms:

- Complex terms are a new kind of individual term (in addition to the **simple individual terms**: individual constants a-v, and the individual variables, w-z)
- Complex terms are of the form $f(t_1, t_2, \ldots t_n)$ where *f* is a **n**-place functor and $t_1, t_2, \ldots t_n$ are individual terms.
- **Open terms** are individual terms that are or that contain variables. **Closed terms** are individual terms that are not or do not contain variables.

Syntax for PLE

- The syntax for PLE is essentially the same as the syntax for PL, except that the identity predicate (=) is added as a predicate in PL and complex terms with functors are added to PL as individual terms.
- See pp. 374-375 for details