Philosophy 220

Truth-Functional Equivalence and Consistency

Review

- Equivalency: The members of a pair of sentences are logically equivalent if and only if it is not (logically) possible for one of the sentences to be true while the other sentence is false.
- Consistency: A set of sentences is logically consistent if and only if it is (logically) possible for all the members of that set to be true at the same time.

Equivalence (Formally):

- Sentences **P** and **Q** of SL are truth-functionally equivalent if and only if there is no truth value assignment [for the components of **P** and **Q**] on which **P** and **Q** have different truth-values.
- This means that on a full truth table, the columns for any two truth-functionally equivalent sentences of SL will be identical.

Finding Equivalence on a shortened truth-table

- As with tautology and contradiction, we test for equivalence by looking for a counterexample.
- If we assume that one of the sentences is true and the other false, then either we will or will not get a coherent truth-value assignment. If we do, then the two sentences are shown not to be equivalent. If we cannot get a coherent truth-value assignment assuming that one sentence is true while the other is false, then we must try it the other way before drawing any conclusions. (why?)

~(B & ~A) and (A v B)

A	B	~	(B &	~A)		A v B

Which columns should be identical if these two sentences are equivalent?

~(B & ~A) and (A v B)

A	B	~	(B &	~A)		A v B

Which columns should be identical if these two sentences are truth-functionally equivalent?

~(B & ~A) and (A v B)

A	B	~	(B &	~A)		A v B
		T				F

So assume that one is T and the other F.

~(B & ~A)   and   (A v B)

A	B	~	(B &	~A)		A v B
		T				F

Note that the only way for A v B to be false is for both A and B to be false.

~(B & ~A)   and   (A v B)

A	B	~	(B &	~A)		A v B
F	F	T				F

Note that the only way for A v B to be false is for both A and B to be false.

~(B & ~A) and (A v B)

A	B	~	(B &	~A)		A v B
F	F	T		T		F

Now we see if this truth-assignment is coherent…

~(B & ~A) and (A v B)

A	B	~	(B &	~A)		A v B
F	F	T	F	T		F

It is coherent. If A and B are both false, then ~(B & ~A) must be true.

F & (J v H) and (F & J) v H

F	H	J	F &	(J v H)		(F & J)	v H

Which columns should be identical if these two sentences are truth-functionally equivalent?

F & (J v H) and (F & J) v H

F	H	J	F &	(J v H)		(F & J)	v H

Which columns should be identical if these two sentences are truth-functionally equivalent?

F & (J v H) and (F & J) v H

F	H	J	F &	(J v H)		(F & J)	v H
			T				F

Assume one is T and the other F…

F & (J v H) and (F & J) v H

F	H	J	F &	(J v H)		(F & J)	v H
	F		T			F	F

If the disjunction '(F & J) v H' is false, then both disjuncts must be false.

F & (J v H) and (F & J) v H

F	H	J	F &	(J v H)		(F & J)	v H
T	F		T	T		F	F

If the conjunction ‘F & (J v H)’ is true then both of its conjuncts must be true.

F & (J v H)   and   (F & J) v H

F	H	J	F &	(J v H)		(F & J)	v H
T	F	?	T	T		F	F

Is there a truth-value assignment for J that is coherent?

F & (J v H) and (F & J) v H

F	H	J	F &	(J v H)		(F & J)	v H
T	F	T	T	T		F	F

FAIL!

Is there a truth-value assignment for J that is coherent?

T is not coherent because it would make (F & J) true, which would in turn make ((F & J) v H) true.

F & (J v H)   and   (F & J) v H

F	H	J	F &	(J v H)		(F & J)	v H
T	F	F	T	T		F	F

FAIL!

Is there a truth-value assignment for J that is coherent?

F is not coherent because it would make (J v H) false, which would in turn make (F & (J v H)) false.

F & (J v H) and (F & J) v H

F	H	J	F &	(J v H)		(F & J)	v H
T	F	X	T	T		F	F

FAIL!

Does this mean that F & (J v H) and (F & J) v H are equivalent?

F & (J v H) and (F & J) v H

F	H	J	F &	(J v H)		(F & J)	v H
T	F	X	T	T		F	F

FAIL!

Does this mean that F & (J v H) and (F & J) v H are equivalent?

IT DOES NOT!

We have shown that F & (J v H) cannot be true while (F & J) v H is false, but it is still possible that F & (J v H) can be false while (F & J) v H is true.

So let's check:

F & (J v H) and (F & J) v H

F	H	J	F &	(J v H)		(F & J)	v H
			F				T

So assume one is F and the other T (the opposite of what we began with)…

F & (J v H)   and   (F & J) v H

F	H	J	F &	(J v H)		(F & J)	v H
F	T		F				T

We have many ways to proceed here, so let us assume that F is false (to make the conjunction it is in false) and that H is true (to make the disjunction that it is in true). If this turns out to be incoherent, there are several other possibilities to try.

F & (J v H) and (F & J) v H

F	H	J	F &	(J v H)		(F & J)	v H
F	T	?	F				T

Now we must see if any truth value of J would yield a coherent table…

F & (J v H) and (F & J) v H

F	H	J	F &	(J v H)		(F & J)	v H
F	T	T	F				T

Let's try T first.

F & (J v H)   and   (F & J) v H

F	H	J	F &	(J v H)		(F & J)	v H
F	T	T	F				T

'J v H' comes out true on this set of assignments while 'F & J' comes out false.

F & (J v H) and (F & J) v H

F	H	J	F &	(J v H)		(F & J)	v H
F	T	T	F	T		F	T

This is a coherent truth-value assignment for F, H, and J that reveals that these two sentences are not truth-functionally equivalent.

The Full Truth-Table (for illustration)

F	H	J	F &	(J v H)		(F & J)	v H
T	T	T	T	T		T	T
T	T	F	T	T		F	T
T	F	T	T	T		T	T
T	F	F	F	F		F	F
F	T	T	F	T		F	T
F	T	F	F	T		F	T
F	F	T	F	T		F	F
F	F	F	F	F		F	F

We proved with our first shortened truth-table that the first sentence is never true while the second sentence is false…

The Full Truth-Table (for illustration)

F	H	J	F &	(J v H)		(F & J)	v H
T	T	T	T	T		T	T
T	T	F	T	T		F	T
T	F	T	T	T		T	T
T	F	F	F	F		F	F
F	T	T	F	T		F	T
F	T	F	F	T		F	T
F	F	T	F	T		F	F
F	F	F	F	F		F	F

…however, we showed with the second shortened truth-table that the two are not equivalent because the second sentence can be true while the first sentence is false.

Consistency

- A set of sentences of SL is consistent if and only if there is at least one truth value assignment [of the constituents of the set of sentences] on which all the members of the set are true.
- That means that if each of the set of sentences of SL were done on a truth-table, there would be one ***row*** of the truth table on which all of the sentences of the set are true.

Shortened tables

- Since a single example of a case in which all of the sentences of a set can be true shows that the set is consistent, when we check for consistency with a shortened truth-table, we should assume that all of the sentences of the set are true. If we get a coherent truth-value assignment from this assumption, then the set is consistent. If we cannot, then the set is inconsistent.
- Checking for counterexample, as we do with tautology, contradiction, and contingency would be going about it the long way.

Is {H ⊃ J, J ⊃ K, K ⊃ ~H} a consistent set?

H	J	K	H ⊃ J		J ⊃ K		K ⊃	~H

Is {H ⊃ J, J ⊃ K, K ⊃ ~H} a consistent set?

H	J	K	H ⊃ J		J ⊃ K		K ⊃	~H
			T		T		T	

Assume that each is true.

Is {H ⊃ J, J ⊃ K, K ⊃ ~H} a consistent set?

H	J	K	H ⊃ J		J ⊃ K		K ⊃	~H
T			T		T		T	

H ⊃ J being true is consistent with several outcomes, so let's assume that H is true to start with.

Is {H ⊃ J, J ⊃ K, K ⊃ ~H} a consistent set?

H	J	K	H ⊃ J		J ⊃ K		K ⊃	~H
T	T		T		T		T	

If H is true, then J must be true to preserve the truth of H ⊃ J.

Is {H ⊃ J, J ⊃ K, K ⊃ ~H} a consistent set?

H	J	K	H ⊃ J		J ⊃ K		K ⊃	~H
T	T	T	T		T		T	

If J is true, then K must be true to preserve the truth of J ⊃ K.

Is {H ⊃ J, J ⊃ K, K ⊃ ~H} a consistent set?

H	J	K	H ⊃ J		J ⊃ K		K ⊃	~H
T	T	T	T		T		T	T

If K is true, then ~H must be true to preserve the truth of K ⊃ ~H.

Is {H ⊃ J, J ⊃ K, K ⊃ ~H} a consistent set?

H	J	K	H ⊃ J		J ⊃ K		K ⊃	~H
T	T	T	T		T		T	T

FAIL!

Failure, H and ~H cannot both be true at the same time.

Is {H ⊃ J, J ⊃ K, K ⊃ ~H} a consistent set?

H	J	K	H ⊃ J		J ⊃ K		K ⊃	~H
F			T		T		T	

Let's try this again…

Assume that each member of the set is true.

Then, since assuming H was true brought us an inconsistent set, let's assume H is false instead.

Is {H ⊃ J, J ⊃ K, K ⊃ ~H} a consistent set?

H	J	K	H ⊃ J		J ⊃ K		K ⊃	~H
F			T		T		T	T

If H is false, then ~H is true.

Is {H ⊃ J, J ⊃ K, K ⊃ ~H} a consistent set?

H	J	K	H ⊃ J		J ⊃ K		K ⊃	~H
F	F	F	T		T		T	T

The truth-value of H ⊃ J is assured by H having a truth-value of false, whatever J's truth-value.

Also, the truth of K ⊃ ~H is assured by ~H being true, whatever K's truth-value.

So we can select any truth values for J and K so long as they don't make J ⊃ K false. (F for both will do)

Is {H ⊃ J, J ⊃ K, K ⊃ ~H} a consistent set?

H	J	K	H ⊃ J		J ⊃ K		K ⊃	~H
F	F	F	T		T		T	T

This is one of several examples of truth-value assignments on which all three sentences end up true, so we have proven that the above set of sentences is consistent.