Philosophy 220

Truth-Functional Entailment and Validity

Entailment

- A set Γ of sentences of SL truth-functionally entails a sentence **P** if and only if there is no truth-value assignment on which every member of Γ is true and **P** is false.
- We use the double-turnstile, '$\models$' to indicate entailment, while we use the negated turnstile, '$\not\models$' to indicate non-entailment.
- Also, notice the use of 'Γ' as a metavariable ranging over sets of sentences of SL.

Checking for Entailment

- On a full truth-table, An entailment relation holds between some set Γ of sentences of SL and **P** if and only if there is no ***row*** of the truth table in which every member of Γ is true while **P** is false.
- A partial truth table can prove non-entailment by arriving at a coherent truth-value assignment while assuming every member of Γ is true while assuming **P** is false.
- If no such coherent truth-value assignment exists, then the entailment relation holds.

Other Notation Issues

- To the left of the entailment symbol is always either a set {…} or a metavariable ranging over sets of sentences of SL.
- To the right of the entailment symbol is always either a sentence of SL or a metavariable ranging over sentences of SL.
- When nothing is to the left of the entailment symbol (as in '⊨ **Q**') it is to be understood that this is shorthand for saying that **Q** is entailed by the empty set, symbolized 'Ø', which is a set that contains no members.

- If $\{P\} \models Q$ and $\{Q\} \models P$, does this mean that P and Q are equivalent?

Self Test 1

- If {P} ⊨ Q and {Q} ⊨ P, does this mean that P and Q are equivalent?
- YES.
- If there are no conditions under which P is true while Q is false, and also no conditions under which Q is true while P is false, then P and Q always have a truth-value in common, and so are truth-functionally equivalent.

Self Test 1

- If $\varnothing \nvdash Q$, what do we know for sure about Q?

Self Test 2

- If $\varnothing \nvdash Q$, what do we know for sure about Q?
- We know that Q is a tautology, because only tautologies are true even when nothing else is.

Self Test 2

- What does Γ entail if it is inconsistent?

Self Test 3

- What does Γ entail if it is inconsistent?
- Any sentence of SL is by any inconsistent set.
- This is because there will never be a case in which all of the sentences in Γ are true, so it will never be the case that all of the members of Γ are true while $\mathbb{P}$ is false.
- This could be called 'trivial entailment'.

Self Test 3

Validity

- Arguments occur when some set of sentences are designated as premises while another sentence is designated as the conclusion.
- Validity is a special case of entailment that applies to arguments.
- An argument is valid if and only if its conclusion is entailed by the set of sentences comprised by its premises.

Validity, Entailment, and the Material Conditional

- If **P**, **Q**, and **R** are each premises, and **S** is the conclusion of a valid argument, then the following entailment relation must hold:
  - $\{P, Q, R\} \models S$
- If that entailment relation holds, then the following material conditional is a tautology:
  - $(P \ \& \ (Q \ \& \ R)) \supset S$