Philosophy 220

Derivations 3

Derivability

- A sentence **P** of *SL* is derivable in *SD* from a set Γ of sentences of *SL* if and only if there is a derivation in *SD* in which all the primary assumptions are members of Γ and **P** occurs within the scope of only the primary assumptions.
- Note that if and only if **P** is derivable from Γ, then $\Gamma \models \mathbf{P}$

Validity

- If the conclusion of an argument is derivable in *SD* from all and only its premises, then the argument is valid.

Theorems

- A sentence **P** of *SL* is a theorem in SD if and only if **P** is derivable in *SD* from the empty set, ∅.
- Since all and only tautologies are entailed by the empty set, all and only tautologies are theorems in *SD*.

Equivalence

- The sentences **P** and **Q** of *SL* are equivalent in *SD* if and only if **P** ≡ **Q** is a theorem in *SD*.

Inconsistency

- A set Γ of sentences of *SL* is inconsistent in *SD* if and only if a contradiction is derivable from Γ.
- A set of sentences of SL is consistent when it is not inconsistent.

Sample Proof of Equivalence:

- The front flap of the book indicates that ‘Q ⊃ P’ is equivalent to ‘~P ⊃ ~Q’.
- This is known as Contraposition.
- If they really are equivalent, then the sentence ‘Q ⊃ P ≡ ~P ⊃ ~Q’ should be a theorem in *SD*.

Derive Q ⊃ P ≡ ~P ⊃ ~Q

1a	1.	Q ⊃ P	A1a / ≡I (~P ⊃ ~Q)
2 1a	2.	~P	A2 / ⊃I (~Q)
3 2 1a	3.	Q	A3 / ~I (~Q)
3 2 1a	4.	P	1,3 ⊃E
3 2 1a	5.	P & ~P	2,4 &I (contra)
! 2 1a	6.	~Q	3-5 ~I
! 1a	7.	~P ⊃ ~Q	2-6 ⊃I
1b	8.	~P ⊃ ~Q	A1b / ≡I (Q ⊃ P)
4 1b	9.	Q	A4 / ⊃I (P)
5 4 1b	10.	~P	A5 / ~I (P)
5 4 1b	11.	~Q	8,10 ⊃E
5 4 1b	12.	~Q & Q	9,11 &I (contra)
! 4 1b	13.	P	10-12 ~I
! 1b	14.	Q ⊃ P	9-13 ⊃I
!	15.	Q ⊃ P ≡ ~P ⊃ ~Q	1-7,8-14 ≡I