Philosophy 220

DERIVATIONS 2

New Assumptions

- Some derivations require that we make assumptions in addition to the assumptions we start with.
- The rules that require additional assumptions are:
- Conditional Introduction ⊃I
- Biconditional Introduction ≡I
- Negation Introduction ~I
- Disjunction Elimination vE

Conditional Introduction ⊃I

- This is also known as 'Conditional Proof'

i	M.	$\mathbb{P}$	Ai
i	N.	$\mathbb{Q}$	-
!	N+1	$\mathbb{P} \supset \mathbb{Q}$	M-N ⊃I

A Conditional Proof:

- We already know that the hypothetical syllogism is valid. Now we may prove it:

$$\begin{array}{l} A \supset B \\ \underline{B \supset C} \\ A \supset C \end{array}$$

A Conditional Proof

- Derive A ⊃ C

	1.	A ⊃ B	Premise
	2.	B ⊃ C	Premise
1	3.	A	A1 /⊃I
1	4.	B	1,3 ⊃E
1	5.	C	2,4 ⊃E
!	6.	A ⊃ C	3-5 ⊃I

QED

Biconditional Introduction ≡I

i	L.	$\mathbb{P}$	Ai
i	M.	$\mathbb{Q}$	-
j	M+1.	$\mathbb{Q}$	Aj
j	N.	$\mathbb{P}$	-
!	N+1.	$\mathbb{P} \equiv \mathbb{Q}$	L-M, M+1-N, ≡I

Notice that this is just two conditional proofs.

Biconditional Proof:

- We know that the biconditional 'A ≡ B' is the same as the conjunction of 'A ⊃ B' and 'B ⊃ A'.
- Now we may prove it.

…feel the beat from the tambourine…

- Derive A ≡ B

	1.	A ⊃ B	Premise
	2.	B ⊃ A	Premise
1a	3.	A	A1a /≡I
1a	4.	B	1,3 ⊃E
1b	5.	B	A1b /≡I
1b	6.	A	2,5 ⊃E
!	7.	A ≡ B	3-4, 5-6, ≡I

Negation Introduction ~I

i	M.	**P**	Ai
i	N.	**Q** & ~**Q**	-
!	N+1	~**P**	M-N ⊃I

This pattern of reasoning is also known as Reductio Ad Absurdum.

If an assumption that you make leads you to absurdity (in this case, contradiction) then that assumption must be false.

Modus Tollens

- Conditional Elimination is essentially modus ponens, but what about modus tollens?
- We can use Reductio to prove modus tollens:

$$\begin{array}{l} A \supset B \\ \sim B \\ \hline \sim A \end{array}$$

MT via Reductio:

- Derive ~A

	1.	A ⊃ B	Premise
	2.	~B	Premise
1	3.	A	A1 /~I
1	4.	B	1,3 ⊃E
1	5.	B & ~B	2,4 &I
!	6.	~A	3-5 ~I

Disjunction Elimination vE

	K.	**P** v **Q**	-
i	L.	**P**	Ai
i	M.	**R**	-
j	M+1.	**Q**	Aj
j	N.	**R**	
!	N+1.	**R**	K, L-M, M+1-N, vD

Constructive Dilemma:

$A \vee D$

$A \supset C$

$D \supset C$

C

This looks to be a valid argument, so lets prove it.

Dirty Deeds Done Dirt Cheap

- Derive C

	1.	A v D	Premise
	2.	$A \supset C$	Premise
	3.	$D \supset C$	Premise
1a	4.	A	A1a /vE
1a	5.	C	2,4 $\supset$E
1b	6.	D	A1b /vE
1b	7.	C	3.6 $\supset$E
!	8.	C	1, 4-5, 6-7 vE

Reiteration (with other assumptions involved)

i…k	M.	**P**	
i…k, l…n	N.	**Q**	
i…k, l…n	N+1.	**P**	M, R

The notation makes this one look more complicated than it is. It simply allows us to take what we already have and add dependency numerals to it to put it where we want it.

We may at any time repeat any line of a derivation as a later line if both of the following are the case:

1) every assumption in force at the earlier line is still in force
2) we add all the additional dependency numerals which are attached to the step immediately before the line we add.