

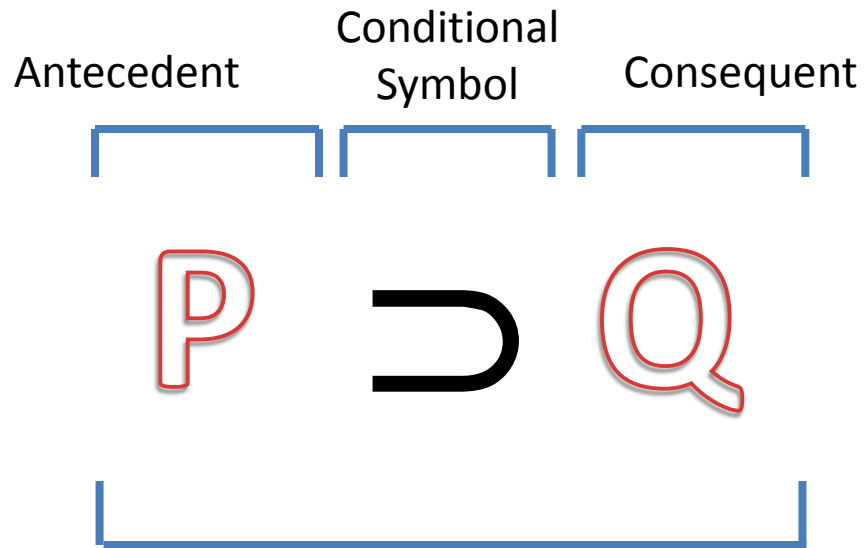
Philosophy 104

6.3 Notes

Recap:

- Propositional Logic is composed of:
 - Propositional Variables
 - Lower-case letters: p, q, r, etc.
 - Truth-Functional Connectives
 - \sim (Negation)
 - $\&$ (Conjunction)
 - \vee (Disjunction)
 - \supset (Material Conditional)
 - Punctuation $[()]$

The Material Conditional



Conditional

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- Consider the truth value of:
 - “If there is an elephant in the room, then it is raining.”

Material Conditional and the English 'If...Then...'

- The Material Conditional is similar to some instances of the English “If...Then...” but not others.
- Consider the truth value of:
 - “If there is an elephant in the room, then it is raining.”
 - It’s TRUE! (as long as there is no elephant in the room, or as long as it is raining)

Material Conditional and the English 'If...Then...'

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- The sentence is true or false based on the truth of its parts and how those parts are connected.

Material Conditional and the English

“If...Then...”

- Consider the truth-value of:
 - “If two plus two is four then Rio de Janeiro is in Chile”

Material Conditional and the English “If...Then...”

- Consider the truth-value of:
 - “If two plus two is four then Rio de Janeiro is in Chile”
 - It’s FALSE
 - Why?

Material Conditional Definition

P	Q	$P \supset Q$
T	T	
T	F	
F	T	
F	F	

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P	Q	$P \supset Q$
T	T	T
T	F	
F	T	
F	F	

Very Straightforward. "If you clean out the barn, then I pay you \$5." is true when it is true that you clean out the barn and true that I pay you \$5.

Material Conditional Definition

P	Q	$P \supset Q$
T	T	T
T	F	F
F	T	
F	F	

Also Straightforward. "If you clean out the barn, then I pay you \$5." is false when it is true that you clean out the barn and false that I pay you \$5. At this point, I have lied to you and you'll want to beat me up.

Material Conditional Definition

P	Q	$P \supset Q$
T	T	T
T	F	F
F	T	T
F	F	T

Now, say you don't clean out the barn. I might decide to give you \$5 out of kindness, in which case I haven't lied to you when I said, "If you clean out the barn, then I pay you \$5."

Material Conditional Definition

P	Q	$P \supset Q$
T	T	T
T	F	F
F	T	T
F	F	T

Say you don't clean out the barn and so I don't give you \$5. I haven't lied to you when I said, "If you clean out the barn, then I pay you \$5."

Other kinds of conditionals

- Remember: “If there is an Elephant in the room, then it is raining.”
 - The above is true (barring an elephant being in the room and clear weather when I present these notes)

Other kinds of conditionals

- Remember: “If there is an Elephant in the room, then it is raining.”
 - The above is true (barring an elephant being in the room and clear weather when I present these notes)
 - If you think it must be false, you are reading it as a causal conditional, which is a material conditional with extra baggage. In a causal conditional “If P then Q” means “P causes Q”.

Other kinds of conditionals

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Other kinds of conditionals

- Consider symbolizing: “If I were you, then I would make popcorn”
 - Notice that there are not *two* propositions expressed because ‘I were you’ does not express a proposition by itself, nor does ‘I *would* make popcorn’.
 - This is a counterfactual, or subjunctive conditional. It is best symbolized ‘*p*’ because it expresses one whole state of affairs.

Material Conditional Equivalence

- Consider whether the following are logically equivalent:
 - “If you clean the barn I’ll pay you \$5.”
 - “Either you don’t clean the barn, or I’ll pay you \$5”

Material Conditional Equivalence

- Consider whether the following are logically equivalent:
 - “If you clean the barn I’ll pay you \$5.”
 - “Either you don’t clean the barn, or I’ll pay you \$5”
- The preceding are symbolized:
 - $C \supset P$
 - $\sim C \vee P$

Material Conditional Equivalence

P	Q	$\sim P$	\vee	Q
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	F

Material Conditional Equivalence

P	Q	$\sim P$	\vee	Q	$P \supset Q$
T	T	F	T	T	T
T	F	F	F	F	F
F	T	T	T	T	T
F	F	T	T	F	T

Material Conditional Equivalence

- Many students want to make a conditional false when the antecedent is false. That would make the symbol ' \supset ' mean the same thing as the '&'.

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- Does 'If P then Q' mean the same thing as 'P and Q'?

Material Conditional Equivalence

- Does 'If P then Q' mean the same thing as 'P and Q'?
- Clearly not. The person who utters the latter is asserting the truth of both P and Q while the person who utters the former is asserting neither the truth nor falsity of either P or Q.

Material Conditional Equivalence

- The material conditional asserts a relationship between P and Q that is false when the antecedent (P) is true while the consequent (Q) is false, and true otherwise.

Exercise XXVII

7. If the house comes up for sale, and I have the money in hand, I will bid on it.

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$$(S \ \& \ M) \supset B$$

Exercise XXVII

8. If you come to dinner, I will cook you a lobster, if you want me to.

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$$D \supset (W \supset L)$$

Exercise XXVII

9. You can be a success if only you try.

10. You can be a success only if you try.

Exercise XXVII

9. You can be a success if only you try.

$T \supset S$

10. You can be a success only if you try.

$S \supset T$

Example: Ex. 24 #5

- Argument is:

p	q	r
---	---	---

$p \supset q$

$q \supset r$

$\sim r$

$\sim p$

- Step 1: Create the reference columns:

Step 2, One column for each
connective:

p	q	r	$p \supset q$	$q \supset r$	$\sim r$	$\sim p$
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Step 3: Fill In Reference Columns

p	q	r	$p \supset q$	$q \supset r$	$\sim r$	$\sim p$
T	T	T				
T	T	F				
T	F	T				
T	F	F				
F	T	T				
F	T	F				
F	F	T				
F	F	F				

Step 4: Fill in remainder

p	q	r	$p \supset q$	$q \supset r$	$\sim r$	$\sim p$
T	T	T				
T	T	F				
T	F	T				
T	F	F				
F	T	T				
F	T	F				
F	F	T				
F	F	F				

Step 4: Fill in remainder

p	q	r	$p \supset q$	$q \supset r$	$\sim r$	$\sim p$
T	T	T	T			
T	T	F	T			
T	F	T				
T	F	F				
F	T	T				
F	T	F				
F	F	T				
F	F	F				

Step 4: Fill in remainder

p	q	r	$p \supset q$	$q \supset r$	$\sim r$	$\sim p$
T	T	T	T			
T	T	F	T			
T	F	T	F			
T	F	F	F			
F	T	T				
F	T	F				
F	F	T				
F	F	F				

Step 4: Fill in remainder

p	q	r	$p \supset q$	$q \supset r$	$\sim r$	$\sim p$
T	T	T	T			
T	T	F	T			
T	F	T	F			
T	F	F	F			
F	T	T	T			
F	T	F	T			
F	F	T	T			
F	F	F	T			

Step 4: Fill in remainder

p	q	r	$p \supset q$	$q \supset r$	$\sim r$	$\sim p$
T	T	T	T			
T	T	F	T	F		
T	F	T	F			
T	F	F	F			
F	T	T	T			
F	T	F	T	F		
F	F	T	T			
F	F	F	T			

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p	q	r	$p \supset q$	$q \supset r$	$\sim r$	$\sim p$
T	T	T	T	T		
T	T	F	T	F		
T	F	T	F	T		
T	F	F	F	T		
F	T	T	T	T		
F	T	F	T	F		
F	F	T	T	T		
F	F	F	T	T		

Step 4: Fill in remainder

p	q	r	$p \supset q$	$q \supset r$	$\sim r$	$\sim p$
T	T	T	T	T	F	
T	T	F	T	F	T	
T	F	T	F	T	F	
T	F	F	F	T	T	
F	T	T	T	T	F	
F	T	F	T	F	T	
F	F	T	T	T	F	
F	F	F	T	T	T	

Step 4: Fill in remainder

p	q	r	$p \supset q$	$q \supset r$	$\sim r$	$\sim p$
T	T	T	T	T	F	F
T	T	F	T	F	T	F
T	F	T	F	T	F	F
T	F	F	F	T	T	F
F	T	T	T	T	F	T
F	T	F	T	F	T	T
F	F	T	T	T	F	T
F	F	F	T	T	T	T

Step 4: Fill in remainder

p	q	r	$p \supset q$	$q \supset r$	$\sim r$	$\sim p$
T	T	T	T	T	F	F
T	T	F	T	F	T	F
T	F	T	F	T	F	F
T	F	F	F	T	T	F
F	T	T	T	T	F	T
F	T	F	T	F	T	T
F	F	T	T	T	F	T
F	F	F	T	T	T	T

Check for Validity:

				Premise 1	Premise 2	Premise 3	Conclusion
	p	q	r	$p \supset q$	$q \supset r$	$\sim r$	$\sim p$
1	T	T	T	T	T	F	F
2	T	T	F	T	F	T	F
3	T	F	T	F	T	F	F
4	T	F	F	F	T	T	F
5	F	T	T	T	T	F	T
6	F	T	F	T	F	T	T
7	F	F	T	T	T	F	T
8	F	F	F	T	T	T	T

meet the MODI

- Modus Ponens

$$\begin{array}{l} p \supset q \\ \underline{p} \\ q \end{array}$$

- Modus Tollens

$$\begin{array}{l} p \supset q \\ \underline{\sim q} \\ \sim p \end{array}$$

Valid Moduses:

Modus Ponens			Modus Tollens				
P2	C	P1			P1	C	P2
p	q	$p \supset q$	p	q	$p \supset q$	$\sim p$	$\sim q$
T	T	T	T	T	T	F	F
T	F	F	T	F	F	F	T
F	T	T	F	T	T	T	F
F	F	T	F	F	T	T	T

A couple common fallacies: (and trouble with conditionals in general)

- Affirming the consequent

$$\begin{array}{l} p \supset q \\ \underline{q} \\ p \end{array}$$

- Denying the antecedent

$$\begin{array}{l} p \supset q \\ \underline{\sim p} \\ \sim q \end{array}$$

Hypothetical Syllogism (Chain Argument)

$p \supset q$

$q \supset r$

$p \supset r$

Valid?

Chain Argument (Hypothetical Syllogism)

			P1	P2	C
p	q	r	$p \supset q$	$q \supset r$	$p \supset r$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T