Philosophy 220

Categorical Logic in PL

Historical Background:

- For about 2500 years, the only formal logic practiced was the logic of categories that was invented by Aristotle.
- In this logic, you would take your natural language sentences, format them as one of four quantified sentences that put things into categories (called the A, E, I, and O claims), and then arrange those claims into a rigidly structured two-premise argument called a syllogism.
- There are several methods of determining when a syllogism is valid (including brute-force memorization of all 16 possible valid syllogism forms)

Historical Background:

- There is nothing essentially wrong with Aristotle's logic (we still teach it in PH104) but it was supplanted by something like PL in the early 20th century because PL is a bit more flexible, versatile, and more compatible with mathematic notation and practice.
- In any case, logicians (like our text authors) still use the names of the classic categorical claims (A, E, I, O) and present their PL counterparts. These four letters were originally the first four Greek vowels, and were transliterated into Roman letters during the middle ages.
- The letter names are arbitrary, so the A-claim has nothing necessarily to do with our symbol '$\forall$' and the E-claim has nothing necessarily to do with our '$\exists$' symbol.

Categorical Statements

- A huge variety of statements in English require us to quantify over something and then put it into a category, including:
  - All dolphins are mammals
  - Some cars are Fords
  - Some horses are not fast runners
  - Only seniors may take this course
  - McDonalds is the only place that serves McNuggets
  - There’s no pleasing some people
  - Etc.
- Despite this variety, each of these sentences is doing one of only four things:

The Four Categorical Forms

	Affirmative	Negative
Universal	(A) All A are B	(E) No A are B
Existential	(I) Some A are B	(O) Some A are not B

Categorical Forms in PL

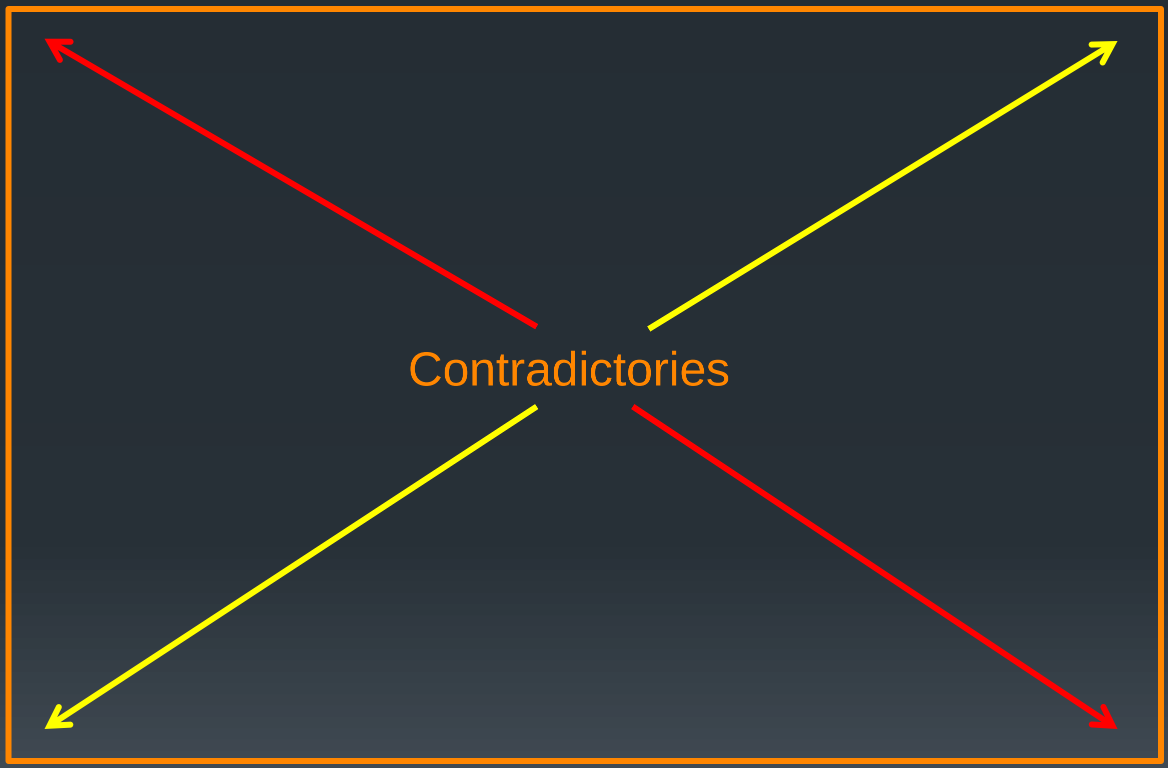
- A claim: All A are B:
  - $(\forall \mathbf{x})(P \supset Q)$
  - $(\forall y)(Ay \supset By)$
- E claim: No A are B:
  - $(\forall \mathbf{x})(P \supset \sim Q)$
  - $(\forall y)(Ay \supset \sim By)$
- I claim: Some A are B:
  - $(\exists \mathbf{x})(P \,\&\, Q)$
  - $(\exists y)(Ay \,\&\, By)$
- O claim: Some A are not B:
  - $(\exists \mathbf{x})(P \,\&\, \sim Q)$
  - $(\exists y)(Ay \,\&\, \sim By)$

The (non)Classical Square of Opposition

A Claim:
$(\forall \mathbf{x})(P \supset Q)$

Contraries

E Claim:
$(\forall \mathbf{x})(P \supset \sim Q)$



I Claim:
$(\exists \mathbf{x})(P \,\&\, Q)$

Sub-Contraries

O Claim:
$(\exists \mathbf{x})(P \,\&\, \sim Q)$

Vocab:

- Contraries cannot be true at the same time (but can be false at the same time).
- Sub-contraries cannot be false at the same time (but can be true at the same time).
- Contradictories cannot have the same truth-value at the same time.

The importance of categorical claims:

- See the example on 319-320
- What these categorical claims allow us to do is to operate using a very broad UD, or even not specify a UD (since we often do not in English).

Consider the Argument:

- Socrates is a man
- Each man is mortal
- Socrates is mortal

- The first premise is an A-claim because claims about particular persons are universal claims (i.e. All things that are Socrates are men)
- The second premise and conclusion are also A-claims, so…

Consider the Argument:

- Socrates is a man $(\forall x)(Sx \supset Mx)$
- Each man is mortal $(\forall x)(Mx \supset Dx)$

- Socrates is mortal $(\forall x)(Sx \supset Dx)$

- Using what we know of the way that truth-functional connectives work, we can see that the above looks valid in PL (as in English).