Philosophy 220

Basic Mechanics of Derivations:

The purpose of a derivation:

- Just as a truth tree checks for consistency, a derivation demonstrates entailment.
- A derivation represents a chain of reasoning that proceeds from a set of assumptions to a final destination.
- Since every step of a derivation is truth-preserving, we can confidently say that in moving from our assumptions to the destination we have preserved truth.

Proving Something

- Derivations are also called 'proofs'
- To prove that an inference is logical, it is sometimes necessary to walk someone through the steps of reasoning that lead from the truth of one set of statements to the truth of another.
- That walk-through is a proof, and the ability to construct a proof is an important concept in thinking logically.

Constructing a derivation:

- Step 1: initial assumptions, goal
- Write down your goal and what you start with. Derive: $B \supset [(\sim R \ \& \sim T) \vee C]$

	1.	(P & Q) & ~R	Assumption
	2.	S & ~T	Assumption

- Number the lines, put in the initial assumptions, and justify each line
- Save the space at the left, you'll need it if you make any additional assumptions.

Step 2: Thinking ahead:

- Look at this derivation, look at your goal, and look at what you have to work with:

Derive: B ⊃ [(~R & ~T) v C]			
	1.	(P & Q) & ~R	Assumption
	2.	S & ~T	Assumption

Thinking ahead:

- Look at this derivation, look at your goal, and look at what you have to work with:

Derive: B ⊃ [(~R & ~T) v C]			
	1.	(P & Q) & ~R	Assumption
	2.	S & ~T	Assumption

The main connective of the goal is a conditional, and it doesn't exist in the initial assumptions, so you now know that the last step of this derivation will use the rule 'Conditional Introduction'

Thinking ahead:

- Look at this derivation, look at your goal, and look at what you have to work with:

Derive: B ⊃ [(~R & ~T) v C]			
	1.	(P & Q) & ~R	Assumption
	2.	S & ~T	Assumption

The way that conditional introduction works is that you assume the antecedent and derive the consequent. Notice that the main connective of the consequent is a disjunction, and there is no disjunction in the initial assumptions , so you will have to use the rule 'Disjunction Introduction'.

Thinking ahead:

- Look at this derivation, look at your goal, and look at what you have to work with:

Derive: B ⊃ [(~R & ~T) v C]			
	1.	(P & Q) & ~R	Assumption
	2.	S & ~T	Assumption

If you have any sentence, you may treat it as one disjunct and introduce any other disjunct. So we should be looking to see if we can get one half of the disjunction in the consequent so that we can get the consequent. Notice that half of the disjunction is indeed in the initial assumptions.

Thinking ahead:

- Look at this derivation, look at your goal, and look at what you have to work with:

Derive: B ⊃ [(~R & ~T) v C]			
	1.	(P & Q) & ~R	Assumption
	2.	S & ~T	Assumption

The parts that we need are parts of conjunctions, and we don't want them to be in the conjunctions that they are currently in. We know we can use the rule 'Conjunction Elimination' to separate conjuncts from their conjunctions. And now we have something we can start doing.

Review of our thought process:

Derive: B ⊃ [(~R & ~T) v C]			
	1.	(P & Q) & ~R	Assumption
	2.	S & ~T	Assumption

1. Separate ~R and ~T from their conjunctions so that we can:
2. Make the disjunction that is the consequent of our target so that we can:
3. Prove that when we assume the antecedent we can end up with the consequent.

So let's go:

- We aim to prove that if '(P & Q) & ~R' and 'S & ~T' are true, then 'B ⊃ [(~R & ~T) v C]' must be true as well (in other words, the first two sentences, as a set, entail the third)
- So we start by assuming the first two sentences are true by writing them on the derivation.
- Now we continue to add lines to the derivation following our rules to make sure that every step of our reasoning is truth-preserving.

Step 3: The Derivation:

Derive: B ⊃ [(~R & ~T) v C]			
	1.	(P & Q) & ~R	Assumption
	2.	S & ~T	Assumption
1	3.	B	A1/⊃I

We start with assuming the antecedent because we want to prove that IF B THEN [(~R & ~T) v C], and if we assume the antecedent, and then end up with the consequent, then we have proven that B ⊃ [(~R & ~T) v C] must be true.

Keep track of assumptions you add to the derivation in the column at left.

Assumptions you make stay in force until they are discharged.
All assumptions must be discharged before the derivation is finished.
The most recent assumption is always discharged first.

The Derivation:

Derive: B ⊃ [(~R & ~T) v C]			
	1.	(P & Q) & ~R	Assumption
	2.	S & ~T	Assumption
1	3.	B	A1/⊃I
1	4.	~R	1, &E

We then separate ~R from its conjunction so that we can later put it into a different one.

If a conjunction is true, then both of its conjuncts are true, so this line is truth-preserving.

Notice A1 is still in force (and will be until it is discharged)

The Derivation:

Derive: B ⊃ [(~R & ~T) v C]			
	1.	(P & Q) & ~R	Assumption
	2.	S & ~T	Assumption
1	3.	B	A1/⊃I
1	4.	~R	1, &E
1	5.	~T	2, &E

We then separate ~T from its conjunction so that we can later put it into a different one.

If a conjunction is true, then both of its conjuncts are true, so this line is truth-preserving.

Notice A1 is still in force (and will be until it is discharged)

The Derivation:

Derive: B ⊃ [(~R & ~T) v C]			
	1.	(P & Q) & ~R	Assumption
	2.	S & ~T	Assumption
1	3.	B	A1/⊃I
1	4.	~R	1, &E
1	5.	~T	2, &E
1	6.	~R & ~T	4,5, &I

We then join ~R and ~T so that we can make the disjunction that is the consequent of our goal.

If two sentences are true separately, then the conjunction of the two of them must be true as well, so this line is truth-preserving.

Notice A1 is still in force (and will be until it is discharged)

The Derivation:

Derive: B ⊃ [(~R & ~T) v C]			
	1.	(P & Q) & ~R	Assumption
	2.	S & ~T	Assumption
1	3.	B	A1/⊃I
1	4.	~R	1, &E
1	5.	~T	2, &E
1	6.	~R & ~T	4,5, &I
1	7.	(~R & ~T) v C	6, vI

We then introduce the disjunction that is the consequent of our goal.

If a sentence is true, then the disjunction of that sentence and any other sentence must be true as well, so this line is truth-preserving.

Notice A1 is still in force (and will be until it is discharged)

The Derivation:

Derive: B ⊃ [(~R & ~T) v C]			
	1.	(P & Q) & ~R	Assumption
	2.	S & ~T	Assumption
1	3.	B	A1/⊃I
1	4.	~R	1, &E
1	5.	~T	2, &E
1	6.	~R & ~T	4,5, &I
1	7.	(~R & ~T) v C	6, vI
!	8.	B ⊃ [(~R & ~T) v C]	3-7, ⊃I

We then introduce the conditional that is our goal.

If we assume the antecedent, and end up with the consequent, using only truth-preserving steps, then we prove that the conditional must be true, so this line is truth-preserving.

Notice A1 is finally discharged because we have met the conditions under which we may introduce the conditional.

Discharging assumptions:

- Making an assumption effectively adds another premise to the proof, so if we end up with our target while some assumption is still in force, we have not proved that the target follows from the initial assumptions, we have proven only that the target follows from the initial assumptions plus some extra ones. So we are not done until all extra assumptions are discharged.
- We make assumptions to prove something, and when the assumption that we make proves what we want it to prove, we discharge it.